1. **Definition:** A subgroup H of a group G is a **normal subgroup** iff $aH = Ha$ for all $a \in G$. We denote a normal subgroup by $H \lhd G$.

2. It is possible for a subgroup H to be normal even if $ah \neq ha$ for all $h \in H$ and for all $a \in G$.

3. **Claim 1:** If G is Abelian, then *every* subgroup H of G is normal.

4. **Claim 2:** Let G be any group. Then $Z(G)$ is normal in G.

5. **Claim 3:** If $|G : H| = 2$, then $H \lhd G$.

   Remember: Having index 2 means H has only two left cosets in G.

6. **Theorem 9.1: A Test for Normality:** A subgroup $H \leq G$ is normal $\Longleftrightarrow (x)^{-1}Hx \subseteq H$ for all $x \in G$.

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