Definition: A subgroup H of a group G is a **normal subgroup** iff $aH = Ha$ for all $a \in G$. We denote a normal subgroup by $H \lhd G$.

Theorem 9.1: A subgroup $H \; \leq \; G$ is normal $\Longleftrightarrow$ $(x)^{-1}Hx \subseteq H$ for all $x \in G$.

Theorem 9.2:

Let $H \lhd G$. The set $G/H = \{aH | a \in G\}$ forms a group under the operation $(aH) * (bH) = (a \circ b)H$, where $\circ$ denotes the group operation in G. G/H is called the **factor group** or **quotient group**.

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Let a be an element of order 24, and consider $G =< a >= \{e, a, a^2, a^3, \ldots, a^{22}, a^{24}\}$. Clearly, $H =< a^6 >$ is a subgroup of G.

1. How do you know G/H is a group?
2. What is $|G/H|$?
3. List the elements of G/H.
4. What is another way of writing $a^{14}H$?
5. What is $a^3H * a^5H$?
6. What is the order of a^2H?

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Let $G = \mathbb{Z}_4 \bigoplus U(4)$, $H =< (2,3) >$, and $K =< (2,1) >$.

1. List the elements of H, and discuss their order.
2. List the elements of K, and discuss their order.
3. Show $H \approx K$.
4. List the elements of G/H, and discuss their order.
5. List the elements of G/K, and discuss their order.
6. Show $G/H \not\approx G/K$.

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