Definition: Let $H \leq G$. For any $a \in G$, the set $aH = \{ah | h \in H\}$ is the **left coset of H in G containing a**. Similarly, $Ha = \{ha | h \in H\}$ is the **right coset of H in G containing a.**

Example: Let $G = \{e, x, x^2, x^3\} =< x >$. Then $H = \{e, x^2\} =< x^2 >$ is a subgroup of G.

On October 23, we found the cosets of H to be:

$$eH = \{e, x^2\} = x^2 H$$
$$xH = \{x, x^3\} = x^3 H$$

$$He = \{e, x^2\} = Hx^2$$
$$Hx = \{x, x^3\} = Hx^3$$

Recall: Just because $aH = bH$ does not mean $ah = bh$ for all $h \in H$. In our above example, $xH = x^3 H$, but $xh \neq x^3 h$ for all $h \in H$. Specifically, $x * e = x^3 * x^2$, $x * x^2 = x^3 * e$.

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