1. Suppose that the order of some finite Abelian group is divisible by 10. Prove that the group has a cyclic subgroup of order 10. Is the same thing true if the order of a finite Abelian group is divisible by 4?

2. How many Abelian groups (up to isomorphism) are there ...

   (a) of order pq where p and q are distinct primes?

   (b) of order pqr where p, q, and r are distinct primes?

   (c) Generalize parts (a) and (b).

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Remember the definition of a group: (Here, I'm denoting the operation by addition for later convenience.)

Def: A group G is a non-empty set with a binary operation (denoted by $a + b$) such that for all $a, b, c \in G$,

1. (Remember the hidden requirement: binary means closed under the operation)
2. $a + (b + c) = (a + b) + c$
3. There is an identity $e \in G$ such that $a + e = e + a = a$ for all $a \in G$. (When the operation is addition, we usually denote the identity by 0, so this becomes "there is a $0 \in G$ such that $a + 0 = 0 + a = a$ for all $a \in G$.)
4. There is an element $-a \in G$ such that $a + (-a) = 0$.

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