

Definition: A subgroup H of a group G is a **normal subgroup** iff $aH = Ha$ for all $a \in G$. We denote a normal subgroup by $H \triangleleft G$.

Theorem 9.1: A subgroup $H \leq G$ is normal $\iff (x)^{-1}Hx \subseteq H$ for all $x \in G$.

Theorem 9.2: Let $H \triangleleft G$. The set $G/H = \{aH | a \in G\}$ forms a group under the operation $(aH) * (bH) = (a \circ b)H$, where \circ denotes the group operation in G . G/H is called the **factor group** or **quotient group**.

November 10, 2004

Let a be an element of order 24, and consider $G = \langle a \rangle = \{e, a, a^2, a^3, \dots, a^{22}, a^{23}\}$. Clearly, $H = \langle a^6 \rangle$ is a subgroup of G .

1. How do you know G/H is a group?
2. What is $|G/H|$?
3. List the elements of G/H .
4. What is another way of writing $a^{14}H$?
5. What is $a^3H * a^5H$?
6. What is the order of a^2H ?

November 10, 2004

Let $G = \mathbb{Z}_4 \oplus U(4)$, $H = \langle (2, 3) \rangle$, and $K = \langle (2, 1) \rangle$.

1. List the elements of H , and discuss their order.
2. List the elements of K , and discuss their order.
3. Show $H \approx K$.
4. List the elements of G/H , and discuss their order.
5. List the elements of G/K , and discuss their order.
6. Show $G/H \not\approx G/K$.

November 10, 2004