Thm 10.1: Properties of Elements under Homomorphisms

Let $\phi : G \to \bar{G}$ be a homomorphism, and let $g \in G$. Then

1. $\phi(e_G) = e_{\bar{G}}$
2. $\phi(g^n) = (\phi(g))^n$ for all $n \in \mathbb{Z}$
3. If $|g|$ is finite, then $|\phi(g)|$ divides $|g|$

Thm 10.2: Properties of Subgroups under Homomorphisms

Let $\phi : G \to \bar{G}$ be a homomorphism, and let $H \leq G$. Then

1. $\phi(H)$ is a subgroup of $\bar{G}$
2. If H is cyclic, so is $\phi(H)$
3. If H is Abelian, so is $\phi(H)$
4. If $H \triangleleft G$, then $\phi(H) \triangleleft \phi(G)$
6. If $|H| = n$, then $|\phi(H)|$ divides n.
7. If $\bar{K}$ is a subgroup of $\bar{G}$, then $\phi^{-1}(\bar{K})$ is a subgroup of G

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1. Let G be a group of permutations. For each $\sigma \in G$, define

$$sgn(\sigma) = \begin{cases} +1 & \text{if } \sigma \text{ is an even permutation,} \\ -1 & \text{if } \sigma \text{ is an odd permutation.} \end{cases}$$

   Prove that sgn is a homomorphism from G to the multiplicative group $\{+1, -1\}$. What is the kernel?

2. Find the kernel of the homomorphism $p : G \oplus H \to G$ by $p(g, h) = g$.

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Thm 10.1: Properties of Elements under Homomorphisms

Let $\phi : G \to \bar{G}$ be a homomorphism, and let $g \in G$. Then

1. $\phi(e_G) = e_{\bar{G}}$
2. $\phi(g^n) = (\phi(g))^n$ for all $n \in \mathbb{Z}$
3. If $|g|$ is finite, then $|\phi(g)|$ divides $|g|$
4. $Ker(\phi)$ is a subgroup of G
5. If $\phi(g) = g'$, then $\phi^{-1}(g') = gKer(\phi)$.

Thm 10.2: Properties of Subgroups under Homomorphisms

Let $\phi : G \to \bar{G}$ be a homomorphism, and let $H \leq G$. Then

1. $\phi(H)$ is a subgroup of $\bar{G}$
2. If H is cyclic, so is $\phi(H)$
3. If H is Abelian, so is $\phi(H)$
4. If $H \lhd G$, then $\phi(H) \lhd \phi(G)$
5. If $|Ker(\phi)| = n$, then ϕ is a n-to-1 mapping from G onto $\phi(G)$.
6. If $|H| = n$, then $\phi(H)|$ divides n.
7. If $\bar{K}$ is a subgroup of $\bar{G}$, then $\phi^{-1}(\bar{K})$ is a subgroup of G
8. If $\bar{K} \lhd \bar{G}$, then $\phi^{-1}(\bar{K}) \lhd G$
9. If ϕ is onto and $Ker(\phi) = \{e\}$, then ϕ is an isomorphism from G to $barG$.

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Remember:

Definition: If $\phi : G \to \bar{G}$ is a group homomorphism, then $Ker(\phi) = \{g \in G | \phi(g) = e_{\bar{G}}\}$.

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