1. Suppose that ϕ is a homomorphism from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{30}$ and that $Ker(\phi) = \{0, 10, 20\}$. If $\phi(23) = 9$, determine *all* elements that map to 9.

2. Prove that $(A \oplus B)/(A \oplus \{e\})$ is isomorphic to B.

3. Suppose that there is a homomorphism ϕ from $\mathbb{Z}_{17}$ to some group, and that ϕ is not one-to-one. Determine ϕ.

4. If ϕ is a homomorphism from $\mathbb{Z}_{30}$ onto a group of order 5, determine the kernel of ϕ.

5. Prove that there is no homomorphism from $\mathbb{Z}_{16} \oplus \mathbb{Z}_2$ onto $\mathbb{Z}_4 \oplus \mathbb{Z}_4$.

6. How many homomorphisms are there from $\mathbb{Z}_{20}$ *onto* $\mathbb{Z}_{10}$? How many are there to $\mathbb{Z}_{10}$? (That is, how many are there that may or may not be onto?)

November 22, 2004