1. **Definition:** A subgroup $H$ of a group $G$ is a **normal subgroup** iff $aH = Ha$ for all $a \in G$. We denote a normal subgroup by $H \triangleleft G$.

2. It is possible for a subgroup $H$ to be normal even if $ah \neq ha$ for all $h \in H$ and for all $a \in G$.

3. **Claim 1:** If $G$ is Abelian, then *every* subgroup $H$ of $G$ is normal.

4. **Claim 2:** Let $G$ be any group. Then $Z(G)$ is normal in $G$.

5. **Claim 3:** If $|G : H| = 2$, then $H \triangleleft G$.

   **Remember:** Having index 2 means $H$ has only two left cosets in $G$.

6. **Theorem 9.1: A Test for Normality:** A subgroup $H \leq G$ is normal $\Longleftrightarrow (x)^{-1}Hx \subseteq H$ for all $x \in G$.

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