Recall from last Friday:

Definition: A **homomorphism** from a group $(G, *)$ to a group $(\bar{G}, \odot)$ is a function $\phi : G \to \bar{G}$ that preserves the group operation. That is,

$$\phi(a * b) = \phi(a) \odot \phi(b).$$

Definition: An **isomorphism** from a group $(G, *)$ to a group $(\bar{G}, \odot)$ is a one-to-one and onto function $\phi : G \to \bar{G}$ that preserves the group action. That is,

$$\phi \text{ is 1-1, } \phi \text{ is onto, and } \phi(a * b) = \phi(a) \odot \phi(b).$$

If an isomorphism from G to $\bar{G}$ (*any* isomorphism) exists, then we say that G and $\bar{G}$ are **isomorphic** and denote this by $G \approx \bar{G}$.

As an example of an isomorphism, we constructed the *canonical* isomorphism from a cyclic group $< a >$ of order 5 and $\mathbb{Z}_5$:

$$\phi :< a >\to \mathbb{Z}_5 \text{ by } \phi(a^k) = k.$$

Find an isomorphism from the group of integers under addition to the group of even integers under addition.

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Show that $U(8)$ is not isomorphic to $U(10)$, but is isomorphic to $U(12)$.

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