Show that $U(8)$ is not isomorphic to $U(10)$, but is isomorphic to $U(12)$.

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1. Let $\mathbb{R}^+$ be the group of positive real numbers under multiplication. Show that the mapping $\phi(x) = \sqrt{x}$ is an automorphism of $\mathbb{R}^+$.

2. Find $\mathrm{Aut}(\mathbb{Z})$.

   Hint: It may be helpful to remember that $\mathbb{Z} = <1>$.

3. Let $r \in U(n)$. Prove that the mapping $\alpha : \mathbb{Z}_n \rightarrow \mathbb{Z}_n$ defined by $\alpha(s) = sr \bmod n$ for all s in Z_n is an automorphism of $\mathbb{Z}_n$