Definition: The **order** of an element $g \in G$ is the smallest positive integer n such that $g^n = e$.

If no such integer exists, then g has **infinite order**. We denote the order of g by $|g|$.

[Remember: $g^n = g * g * g * \ldots * g$. If the operation is addition, we would therefore mean $g + g + \ldots + g = ng$. Notice that $n \notin G$, and that here multiplication isn't the group operation. It's *just* notation.]

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1. $U(10) = \{1, 3, 7, 9\}$, under $\times mod\, 10$.

    (a) What is $|U(10)|$?

    (b) Find the order of each element.

2. $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, under $+$ mod 6.

    (a) What is $|\mathbb{Z}_6|$?

    (b) Find the order of each element.

3. $\mathbb{Z}$, under addition

    (a) What is $|\mathbb{Z}|$?

    (b) Find the order of each element.

Remember: you're paying attention to what goes on when you calculate the order of an element, noticing anything you can about order of element and order of group, and generally speculating as to why we use the same word for these two different ideas.