The Fundamental Theorem of Cyclic Groups:

1. Every subgroup of a cyclic group $\langle a \rangle$ is cyclic.

2. If $|<a>| = n$, then the order of every subgroup of $<a>$ divides n.

3. For each divisor k of n, there is exactly one subgroup of order k, namely $<a^{n/k}>$.

Recall:

Theorem 4.1

Let G be a group, and let $x \in G$.

1. If $|x| = \infty$, then all distinct powers of a are distinct group elements of G.

2. If $|x| = k < \infty$, then $\langle x \rangle = \left\{ e, x, x^2, \ldots, x^{k-1} \right\}$. Moreover,

$$x^i = x^j \iff k \text{ divides } i - j.$$

List all the elements of order 8 in $\mathbb{Z}_{8,000,000}$. How do you know your list is complete?

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