1. You want to paint a picture on a rectangular canvas, and you want the ratio of the length of the long side to the length of the short side to be φ, the Golden Ratio – that is, you want the canvas to be in the shape of a *Golden Rectangle*. If the short side of your canvas is 2' wide, exactly how long should you (try to) make it be?

2. Wikipedia says that Salvador Dali "explicitly used the golden ratio in his masterpiece, *The Sacrament of the Last Supper* [shown below]. The dimensions of the canvas are a golden rectangle. " This painting is 267 cm wide and 166.7 cm tall.

   (a) Find an acceptance range for the Golden Ratio, assuming the measurements are each accurate to within 0.5%. (You may assume that accuracy in measurements within 0.5% translates to accuracy in the ratio within 1%.)

   (b) Based on this margin of error and the resulting acceptance range found in part (a), do the measurements support this claim?

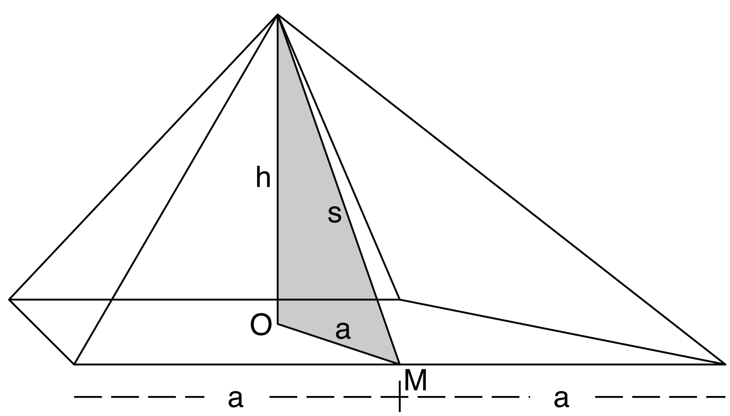
   (c) What do you conclude from this?

   Note: We know that pentagons give rise to the Golden Ratio exactly. Notice that a huge dodecahedron (which is composed of pentagons) is suspended above and behind Jesus. Wikipedia also says that the edges of it appear in golden ratio to one another. Check it out if you like!

3. You have read, in Section 2.2.3, that it is frequently said that Herodotus described the construction of the Great Pyramid by saying that the Pyramid was built so that the area of each face would equal the area of a square whose side is equal to the Pyramid's height. You have also read that this relationship would ensure that the Golden Ratio appear in the Great Pyramid, but I left out some details about *why* this is true. You will work through those details in this exercise.

   Let

$$\begin{aligned} h &= \text{height of pyramid} \\ a &= \frac{1}{2}(\text{length of base}) - \text{so } 2a = \text{length of base} \\ s &= \text{slant height} = \text{the height of a triangular face} \end{aligned}$$



   (a) Find the area of a square whose sides all have length h (the height of the pyramid).

   (b) Find the area of one of the triangular faces of the Pyramid.

   Remember: Area of a triangle=$\frac{1}{2}$*(base)* $\times$ *(height).*

   (c) Rewrite the statement attributed to Herodotus, using the expressions for area you found in parts 3a and 3b.

(d) By looking at the above diagram of the pyramid, find another equation that connects h, s, and a.

Hint: Look for a different triangle!

(e) Combine these two equations in a logical way to find a relationship between a and s. Solve for s/a. (You should get that $s/a = \varphi$!)

4. You have also read in Section 2.2.3 that if the Egyptians used rollers to measure the length of the base of the pyramid, and ropes to measure the height of the pyramid, then π would have been sure to appear in the Great Pyramid. I again left the details to this exercise.

(a) Suppose you build a model of a pyramid as follows: take a wheel of diameter d and lay out a square base whose sides are each one revolution of the wheel long. Then make the pyramid height equal in length to two diameters of the wheel.

i. How long is the base of your model? (Your answer will be in terms of d.)

Remember: Circumference of a circle $= 2\pi\times$ *radius* $= \pi \times$ *diameter.*

ii. How tall is your model? (Again, your answer will be in terms of d.)

iii. Find the ratio of the height of your model to the length of the base of your model.

iv. Find the ratio of the height of the Great Pyramid to the length of the side of the base of the Great Pyramid.

Recall: The height of the Great Pyramid is 481.4 feet and the length of the side of the base of the Great Pyramid is 755.79 feet.

v. Draw some conclusions about the shape of the Great Pyramid and the shape of your model.

(b) In this part of the problem, you're going to show that the Egyptians wouldn't have had to use a gigantic measuring wheel for this process to have worked.

i. Suppose you lay out a square base whose sides are each 10 revolutions of the wheel long, and you make the height be 20 diameters of the wheel. Find the ratio of the height of this new model to the base of this new model. How does it compare to the ratios you found in the previous parts?

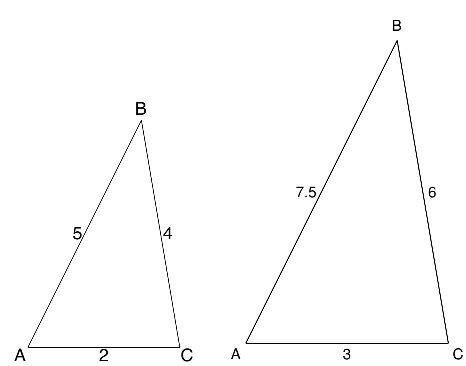
ii. Suppose you lay out a square base whose sides are each n revolutions of the wheel long, and you make the height be $2n$ diameters of the wheel tall. Again, find the ratio of the height of this new model to the base of this model, and compare.

(c) Using the dimensions for the Great Pyramid given above, find the diameter of the measuring wheel required so that 100 revolutions of the wheel would produce one side of the base of the Great Pyramid and 200 diameters would give the height. Is this a reasonable sized for the measuring wheel? That is, is it likely the Egyptians would use a measuring wheel this size, if they constructed the pyramid this way?

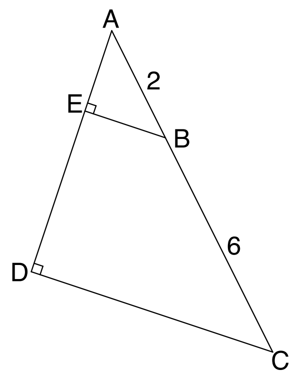
5. **Which are similar**

(a) Which of the following pairs of figures are similar? If they are similar, explain why.

i. the two triangles below:

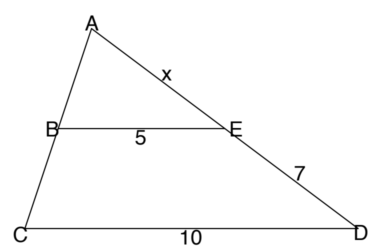


ii. The pair below consists of the big triangle and the smaller one inside it.



(b) For the pair(s) above that you decided were similar, find the scale factor of the sides.

6. Assume that the following pair of triangles are similar, and find the unknown value x.



7. P and Q, shown below (but not to scale), are similar polygons. If the perimeter of P is 10, what is the perimeter of Q?