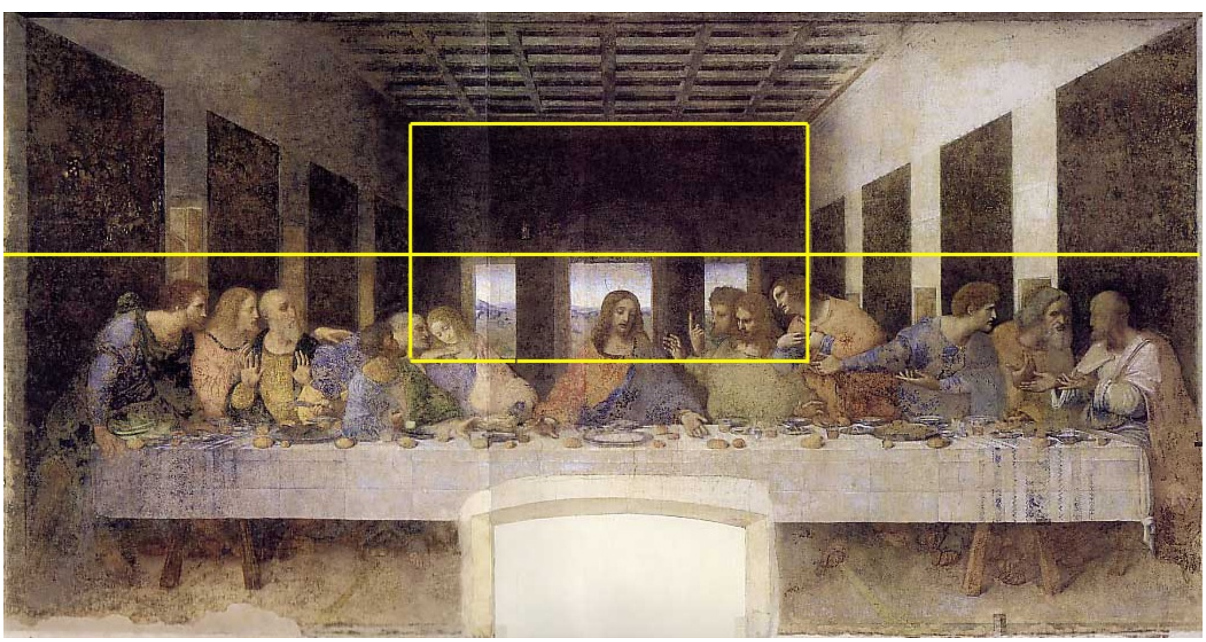
1. In this problem, you will be studying Leonardo da Vinci's *Last Supper*, and (ultimately) deciding whether you believe he was intentionally trying to incorporate the Golden Ratio. Measure as carefully as you can.

   (a) Find the acceptance range for φ based on a 2% margin of error in measurement (which will allow for a 4% margin of error in your ratios).

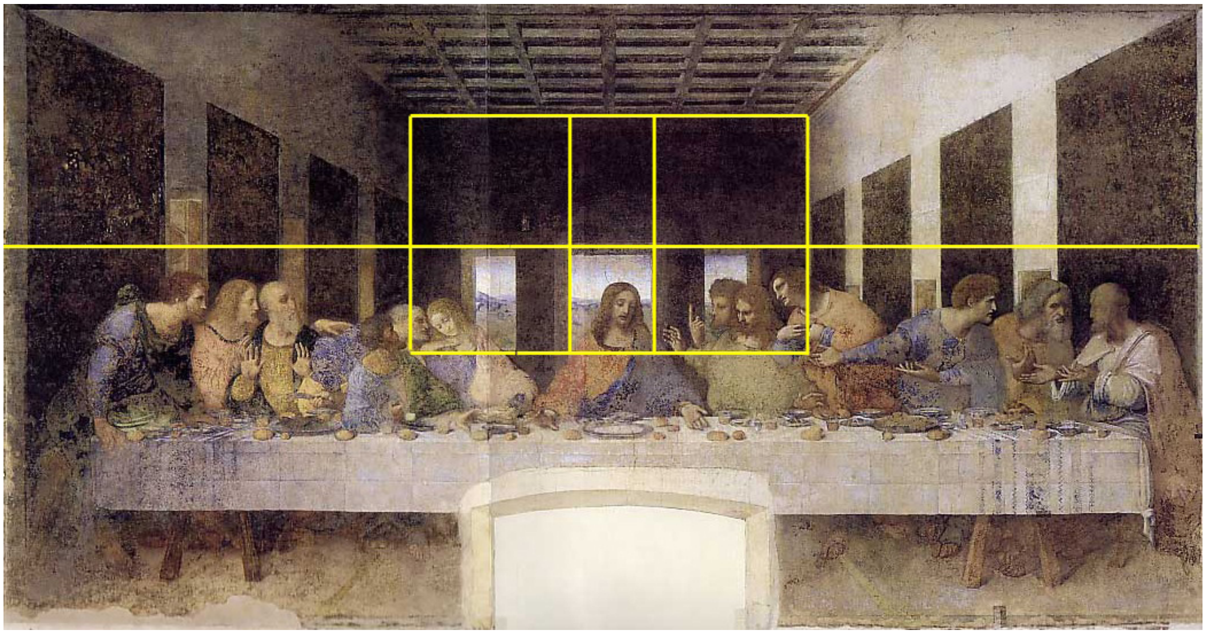
   (b) Below, I have super-imposed a horizontal line that follows the tops of the insides of the windows on the rear wall all the way to the edges of the mural. Does that line cut the height of the painting into Extreme and Mean Ratio?



(c) In the figure on the previous page, I have also outlined the rectangle formed by the rear wall, down to the bottoms of the windows. Is it a Golden Rectangle (within our margin of error)?

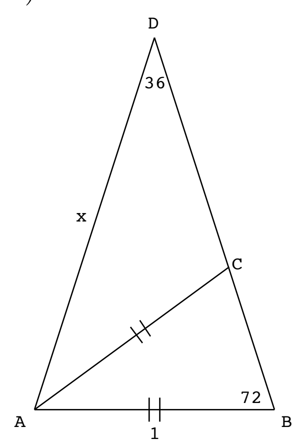
(d) Also in that figure, does the long horizontal line cut the height of the rear-wall rectangle in Extreme and Mean Ratio?

(e) On the next page, I have super-imposed two vertical lines that upwardly extend the vertical lines that define the central window, forming rectangles on the left and right that are subdivided into a top part and a smaller rectangle below. Carefully measure both of the larger rectangles (the ones that go from the top of the wall down to the bottom of the windows, and from the edges of the walls to the window behind Jesus), and determine whether they are Golden, within our acceptance range.



(f) In the same figure, carefully measure both of the smaller rectangles (to the left and right of the central window), and determine whether they are Golden (within our margin of error).

2. *Golden Triangles:* In this exercise, you will be showing that all of the triangles shown in the triangle below have the ratio *long side*:*short side* in the Golden Ratio. Since any triangles with the same angles will be similar, and hence have sides in the same proportions, this will show that any 72°-72°-36° triangle and any 36°-36°-108° triangle will be *Golden*, although you will of course first have to discover those angles. (Also, any isosceles triangle whose sides have this ratio will be similar to one of these two types.)



(a) Show that triangle ABD is similar to triangle BCA.
Hints:
- Since you're given that $\overline{AB} = \overline{AC}$, you know that triangle BCA is an isosceles triangle.
- Remember that the base angles of an isosceles triangle are equal, *and* if the base angles of a triangle are equal, it's an isosceles triangle.
- Although triangle ABD *looks* like an isosceles triangle, you are not given that it is – if you want to use that it is, you need to show it.
- Also remember that the sum of the angles in a triangle are 180°.

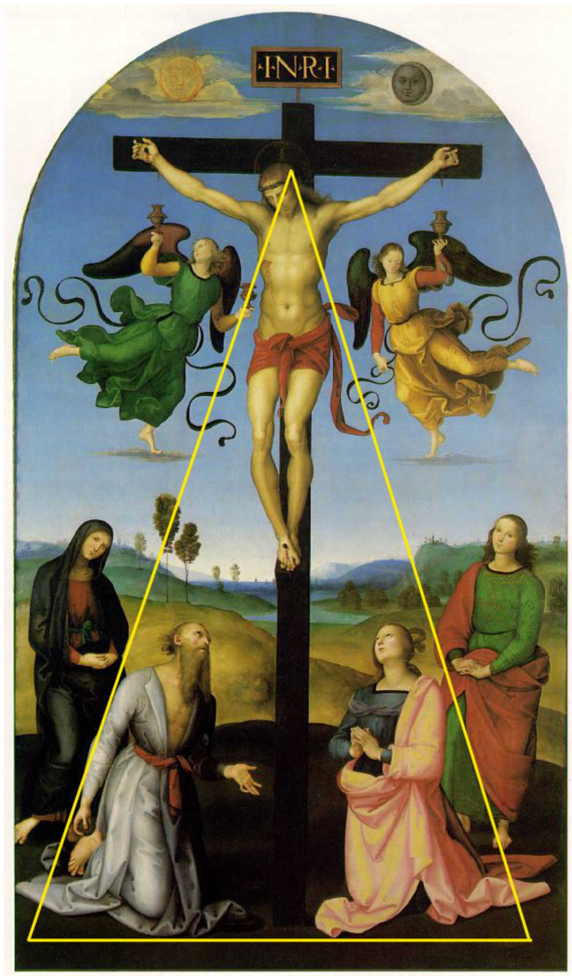
(b) Show that triangle ADC is also an isosceles triangle.

(c) Use the result of part (b) to write the length $\overline{BC}$ in terms of x.

(d) Use the similarity you showed in (a), and your result to part (c), to show that $x = \varphi = \dfrac{1+\sqrt{5}}{2}$.

(e) Show that in the isosceles triangle ADC, the ratio of the longer side to the shorter side is again φ.

3. I found a couple of websites that claim that Raphael's *Mond Crucifixion* exhibits a Golden Triangle. Below is a replica of the diagram I found on the web.



(a) By carefully measuring this diagram, find the ratio of the length of the long side of this triangle to the base.

(b) Assuming a 2% margin of error in your measurements, does this triangle fall within the margin of error being a Golden Triangle?

(c) Do you think the placement of this triangle is defined in a natural or obvious way by the features of the painting, or do you think it was conveniently chosen to create a nearly golden triangle? In case the lines obscure some of the detail: the top vertex is at the point where the horizontal line defined by the bottom of the crossbar intersects the midline defined by Jesus' body; this point is also the rightmost edge of the crown of thorns. The left side of the triangle extends to follow the arm of St. Jerome (kneeling), while the right side extends to follow the cloak of John the Evangelist (standing). The horizontal leg just grazes the robe of St. Jerome on the left and the cloak of Mary Magdalene on the right.

4. We know that the first 10 Fibonacci numbers are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. Remember that we use the notation F_n to represent the nth Fibonacci number – that is, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc. Also remember that we find the nth Fibonacci number by adding together the two that come before it. Find the numerical value of the following:

   (a) F_{11}

   (b) $F_{11} + 2$

   (c) F_{11+2}

5. Given that $F_{36} = 14,930,352$ and $F_{37} = 24,157,817$, find:

   (a) F_{38}

   (b) F_{35}

6. Let a represent the 1000th Fibonacci number and b represent the 1001st Fibonacci number. Express the 1003rd Fibonacci number in terms of a and b. (In other words, you're doing this without ever knowing what the 1000th and 1001st Fibonacci numbers are.) Simplify your answer.

7. Binet's Formula defines Fibonacci numbers directly:

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}.$$

Use Binet's Formula to find the 15th Fibonacci number.

Hint: Don't try to do this by hand - -use a calculator; specifically, use the kind of calculator that allows you enter a long expression with parenthesese. Be very careful with your parentheses! If you do it all on the calculator (that is, without writing down rounded intermediate results and then re-entering those values later), you *should* end up with a whole number, with no rounding necessary, which always seems miraculous!

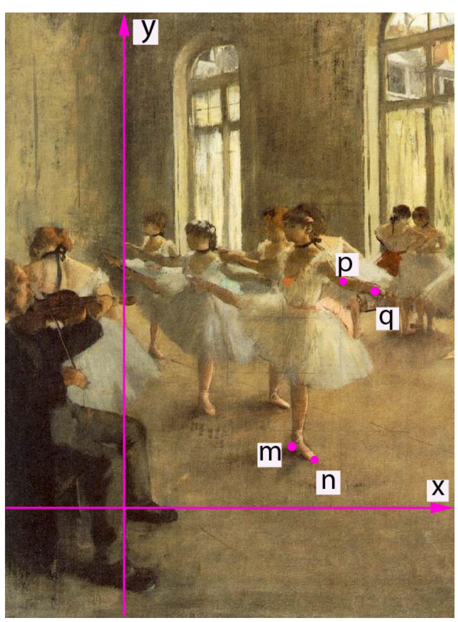
Write down here exactly what you entered in your calculator, with parentheses and all, and then what your result was.

8. Below is a detail from Edgar Degas' painting, *The Rehearsal*, with 2-dimensional coordinate axes superimposed on the picture plane. Using the points $m(216, 88)$, $n(249, 68)$, $p(283, 302)$, and $q(317, 293)$ in the figure (the coordinates are in pixels), find the following distances:

   (a) $d(m, n)$

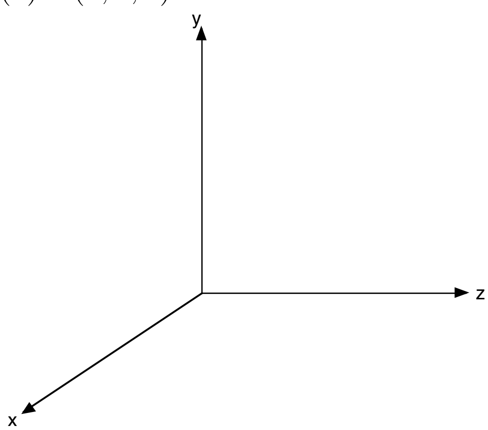
   (b) $d(p, q)$

   (c) Several systems of proportions dictate that a person's foot should be about the same length as their forearm. Thinking of the painting as a window onto a "real" dance studio, it looks as if the actual three-dimensional dancer's left foot and left forearm would be roughly parallel and directly above one another. We'll see later that because of that, if they were indeed the same length in real life, then their *images* in the painting would also be the same length. *Are* these images the same length?

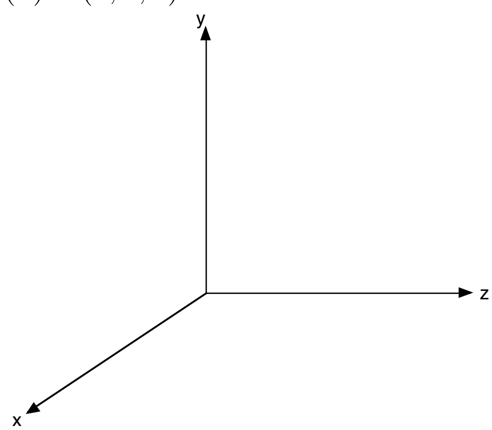


9. Please plot the following points on a set of 3-D coordinate axes (using the coordinate system used for this course rather than the standard coordinate system.) Mark units on your axes, and show enough dashed lines so I can see how you found where to put your points.

(a) $A(1, 3, 4)$



(b) $B(2, 4, 0)$



(c) $C(0, 3, -1)$