You may turn in the homework on the print-out of the assignment, if you like, but turn in a clean copy, not your notes and scratch work.

Part I

The exercises in this part of the problem set deal with a point P that moves farther and farther away from the picture plane and the viewer, without moving left or right, up or down. That is, the x- and y- coordinates of P do not change (they are equal to 2 and 3, respectively), but its z-coordinate gets bigger and bigger.

Throughout these problems, use that the viewer is located on the negative z-axis, the viewing distance d is 5 units and use the Perspective Theorem.

1. Suppose $P = (2, 3, 5)$. What are the values of x' and y', the coordinates of the perspective image of P?

2. Now suppose $P = (2, 3, 95)$. What are x' and y'?

3. What if $P = (2, 3, 995)$?

4. Draw one TOP VIEW and one SIDE VIEW like those we did in class (they're also in Figure 2 of Lesson 2 in *Lessons in Mathematics and Art*), and include the viewer's eye and all the points P and P' from parts (a)-(c), along with light rays from those points to the viewer's eye (the drawings need not be to scale). Can you see what's happening?

5. Consider a point $P(x, y, z)$. If x and y do not change, but z gets bigger and bigger, what happens to the picture plane image P' of P? Use your observations from parts (a)-(d).

6. Our everyday experience tells us that objects appear smaller as they get farther away. Explain how this is consistent with your answers to parts (a)-(e).

Part II

In this portion of the assignment, you are going to draw the same cube in different positions, using the Perspective Theorem. You will then use these pictures to make observations that should reinforce the conclusions about the perspective images of various types of lines that are being discussed in class. You *will* need graph paper! (I would suggest having each unit be several squares long, so your pictures are big enough to really appreciate.)

Note: Since you are going to be making observations based on your final graphs, precision in graphing will be important. Connect the dots with a straight edge. You *will* be graded on accuracy and neatness.

In both cases, our cube will have side of length 4, and the viewing distance d (how far the viewer's eye is from the picture plane) will be 8.

We will call our cube $ABCDEFGH$ with the base being square $ABCD$ and the top being square $EFGH$. (Note that in the base square, A is connected to B and D, B is connected to A and C, etc; and that E is directly above A, F is directly above B, etc).

1. What are the coordinates for the viewer's eye?

2. We'll begin with a cube whose top and bottom are horizontal and whose front and back are parallel to the picture plane. The bottom will be above the viewer's eye.

   Use the following coordinates for the corners of the cube:

Base=ABCD		Top=EFGH	
A	$(8,3,4)$	E	$(8,7,4)$
B	$(12,3,4)$	F	$(12,7,4)$
C	$(12,3,8)$	G	$(12,7,8)$
D	$(8,3,8)$	H	$(8,7,8)$

(a) Using the Perspective Theorem, find the coordinates for each of the 8 corners of the image in the picture plane (that is, find (x', y')). You may do these calculations by hand or, if you're comfortable with it, you may use a spreadsheet like Excel. If you do it by hand, include your work on a separate sheet; if you use a spreadsheet, please include it with your work.

(b) *Carefully and precisely* plot the points you found in part 2a in the xy plane on graph paper. (Remember you are *not* using 3D space axes for this!) Then (paying attention to the right order), neatly connect them with straight lines (use a straight edge, and use dashed lines to indicate the hidden edges) to obtain the perspective image.

(c) Get a good idea of what the viewing distance is in the scale you used (that is, how long is 8 units?), and then put one eye at that distance from the page, directly opposite the origin. Look at your perspective image with that one eye. Do you see a cube, with the illusion of depth?

(d) Your cube has one set of four parallel lines which are not parallel to the picture plane. Do those lines look parallel in the perspective image you've created? Using however many straight edges (pieces of paper, for instance) you need, see where they intersect (this may not be on your piece of graph paper). What can you say about where these four lines intersect?

3. We'll continue with the same cube, but we'll turn it so that while the top and bottom are still horizontal, now one edge is facing us, rather than the front and back being parallel to the picture plane. We'll also move it so that the top is below the viewer's eye.

   Use the following coordinates for the corners of the cube:

Base=ABCD		Top=EFGH	
A	$(0, -6, 4)$	E	$(0, -2, 4)$
B	$(2.8, -6, 6.8)$	F	$(2.8, -2, 6.8)$
C	$(0, -6, 9.7)$	G	$(0, -2, 9.7)$
D	$(-2.8, -6, 6.8)$	H	$(-2.8, -2, 6.8)$

(a) Find the coordinates for each of the corners of the image in the picture plane (include your work or spreadsheet). *Carefully and precisely* plot them in the xy plane on graph paper, then neatly connect them with straight lines to obtain the perspective image.

(b) As with the previous exercise, put one eye at the viewing distance opposite the origin. Look at your perspective image with that one eye. Does it leap off the page at you?

(c) Again, can you get a sense of where the parallel lines intersect?