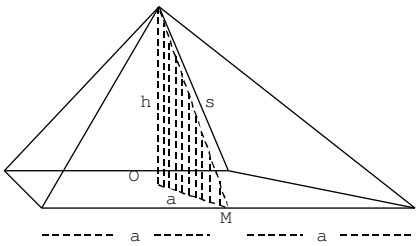
1. You have read, in "Under the Starry Pointed Pyramid", Chapter 3 from Mario Livio's *The Golden Ratio*, that it is frequently said that Herodotus described the construction of the Great Pyramid by saying that the Pyramid was built so that the area of each face would equal the area of a square whose side is equal to the Pyramid's height.

   You have also read that if Herodotus had indeed said this, it would have meant that the Golden Ratio was sure to appear in the Great Pyramid, but Livio left out some details.

   In this exercise, we will explore this idea in more detail.



   (a) Using h for the height of the pyramid, as shown in the above figure, what would be the area of a square whose side is equal in length to the Pyramid's height?

   (b) What is the area of one of the triangular faces of the Pyramid? Use a for the length of half the base, and s for the "slant-height" (the height of a face, as measured on the face), as is used in the above figure.

   (c) Rewrite the statement attributed to Herodotus, using the expressions for area you found in the previous two exercises.

   (d) By looking at the above diagram of the pyramid, find another equation that connects h, s, and a.

(e) Combine these two equations in a logical way to find a relationship between a and s. Solve for s/a. (You should get that $s/a = \varphi$!)

2. You have also read, in the same chapter, that if the Egyptians used roller drums to measure the length of the base of the pyramid, and palm fiber ropes to measure the height of the pyramid, then π would have been sure to appear in the Great Pyramid. Again, Livio left out some details, so in this exercise, we will explore in more detail.

   (a) Suppose you take a wheel of diameter d and lay out a base whose sides are each one revolution of the wheel long. Then make the pyramid height equal in length to two diameters of the wheel. Show that this pyramid has (almost) the exact same shape as the Great Pyramid. (This will involve proportions.)

   (b) Show that the Egyptians wouldn't have had to use a gigantic measuring wheel for this process to have worked. That is, show that as long as the number of diameters you use to make the height is exactly twice the number of revolutions you use to make the side of the base, you get the same shaped pyramid.

   (c) The height of the pyramid is 481.4 feet, and the average length of each of the four sides is 755.79 feet. Find the diameter of the measuring wheel required so that 100 revolutions of the wheel would produce one side of the base of the Great Pyramid and 200 diameters would give the height. Is this a reasonable sized for the measuring wheel? That is, is ti likely the Egyptians would use a measuring wheel this size, if they constructed the pyramid this way?

3. Spend the next couple of days with ruler in hand, measuring all the rectangles around you that you can. See if you can find any (nearly) Golden Rectangles! Is your window a golden rectangle? Your textbook from some class? The front or side of that box of macaroni and cheese you make for a midnight snack? Your credit card? A box of cigarettes (not that I'm advocating buying or smoking them)? Your bed?

   For each item you investigate, write down what it is, give the measurements, and include your calculation of the ratio. Use our acceptance interval of 1.585 to 1.651.

(Don't bother including measurements for rectangles that are obviously not golden just from looking at them – if a rectangle is obviously twice as wide as it is high, there's no real point in measuring).