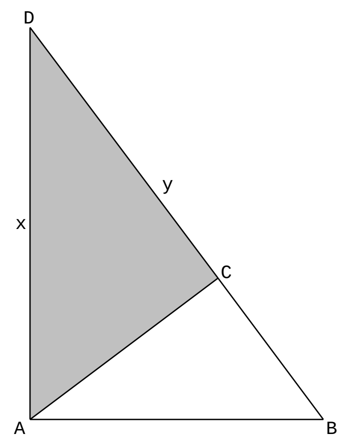
1. Rectangle A is 10 by 20. Rectangle B is gnomon to rectangle A. What are the dimensions of rectangle B?

2. Find the values of x and y so that the shaded triangle is a gnomon to the white triangle ABC.

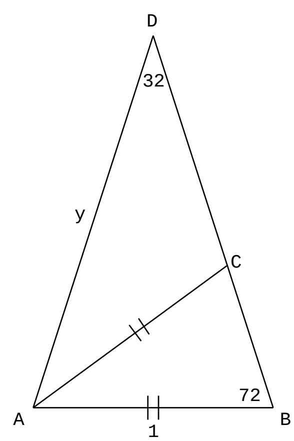


3. A rectangle has a 10 by 10 square gnomon. What are the dimensions of the rectangle?

4. A rectangles has a square gnomon. The new rectangle obtained by attaching the square gnomon to the original rectangle has longer leg 20. What are the dimensions of the original rectangle?

5. Sometimes, when reading about how the Golden Ratio appears in Art, you'll read about Golden Triangles. In this exercise, we'll learn what those are.

   In the figure below, triangle BCD is a $72° - 72° - 36°$ triangle with base of length 1 and the longer sides of length x.
   Notice: Using this choice of values, the ratio of the longer side to the shorter side is $\frac{x}{1} = x$.



   (a) Show that triangle ABD is similar to triangle BCA.

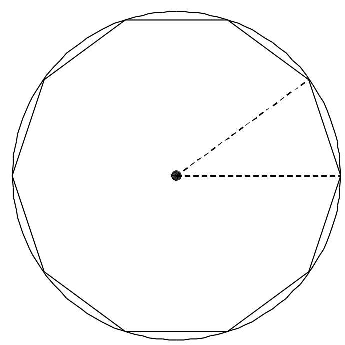
   (b) Use the similarity you showed in (a) to show that $x = \varphi = \frac{1+\sqrt{5}}{2}$.

   (c) What are the interior angles of triangle ACD?

   (d) Show that in the isosceles triangle ACD, the ratio of the longer side to the shorter side is again φ.

6. We saw in class that $\frac{4}{\pi}$ is amazingly close to $\sqrt{\varphi}$. I speculated that there was probably some geometric figure inscribed in a circle that produces this relationship between these two remarkable constants, but I didn't know what it was. Well, I *still* don't know what the connection is, but I did find another relationship between φ and π, and in this exercise, we're going to explore it. ...

   A regular decagon (that is, a figure with 10 equal sides and 10 equal angles) can be inscribed in a circle of radius r, as shown below. Using $r = 1$, to make the calculations simpler,



   (a) find the perimeter of the decagon in terms of φ, using the results of the previous problem.

   (b) use that the perimeter of the decagon and the circumference of the circle are roughly equal to find an approximate expression that relates φ and π.

7. We know that the first 10 Fibonacci numbers are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. Remember that we use the notation F_n to represent the nth Fibonacci number – that is, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc. Find the numerical value of the following:

   (a) F_{11}

   (b) $F_{11} + 2$

   (c) F_{11+2}

8. Given that $F_{36} = 14,930,352$ and $F_{37} = 24,157,817$, find:

   (a) F_{38}

   (b) F_{35}

9. Remember the amazing Binet's formula, which allows us to find F_N without having to first find the first $N-1$ Fibonacci numbers, and which accomplishes that by bringing φ in to the mix:

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)}{\sqrt{5}}.$$

   Verify that Binet's Formula works for $N = 3$.

10. As you can see from using Binet's formula yourselves, using Binet's formula before the advent of good calculators would be a pain, and using it before calculators were invented would have seemed extremely difficult. Does that mean that Binet's Formula was essentially useless back when Euler invented it around 250 years ago? No! In this exercise, we'll see an easier way to calculate powers of φ.

    Remember that φ is one of two solutions to $x^2 - x - 1 = 0$ ($\frac{1-\sqrt{5}}{2}$ is the other). So $\varphi^2 - \varphi - 1 = 0$, or $\varphi^2 = \varphi + 1$.

    (a) Show that $\varphi^3 = 2\varphi + 1$.

    (b) Show that $\varphi^4 = 3\varphi + 2$.

    (c) Show that $\varphi^5 = 5\varphi + 3$.

    (d) Look at the results for φ^2, φ^3, φ^4, and φ^5. Based on what you see, what do you think φ^6 is ? Check your results.

    (e) In general, how do you think φ^N can be rewritten, in terms of just a single power of φ and some whole numbers?

11. Please plot the following points on a set of 3-D coordinate axes. (You may want to use graph paper for this!)

    (a) $A(1,3,4)$

(b) $B(2, 4, 0)$

(c) $C(0, 3, -1)$

12. In this problem, you're going to be considering a box whose faces are parallel to the coordinate planes. Suppose the corners of this box have the following coordinates:

Bottom	Top
$A(1, 3, 4)$	$E(1, 7, 4)$
$B(8, 3, 4)$	$F(8, 7, 4)$
$C(8, 3, 10)$	$G(8, 7, 10)$
$D(1, 3, 10)$	$H(1, 7, 10)$

(a) How wide is the box in the x direction? '

(b) How tall is the box in the y direction?

(c) How deep is the box in the z direction?

13. Now I want you to look at the patterns you may have noticed in the first problem, and use them. We again have a box whose faces are parallel to the coordinate planes. Suppose the coordinates of two opposing corners of a box (the front-left-bottom corner and the rear-right-top corner) have coordinates $(3, -1, 5)$ and $(10, 7, 10)$.

(a) How wide is the box in the x direction?

(b) How tall is the box in the y direction?

(c) How deep is the box in the z direction?

(d) What are the coordinates of the remaining 6 corners?