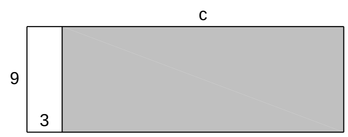
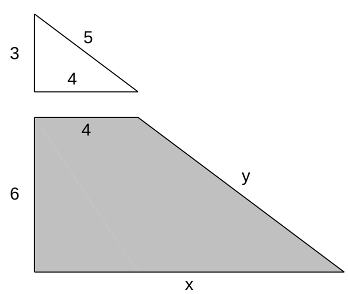
Note: The first twelve problems are all from *Excursions in Modern Mathematics*, by Tannenbaum and Arnold.

1. Find the length of c of the shaded rectangle so that it is a gnomon to the white rectangle with sides 3 and 9.

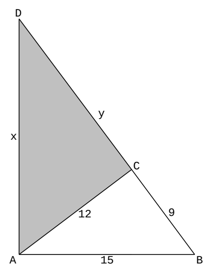


2. Find the values of x and y so that the shaded figure is a gnomon to the white triangle.



3. Rectangle A is 10 by 20. Rectangle B is gnomon to rectangle A. What are the dimensions of rectangle B?

4. Find the values of x and y so that the shaded triangle is a gnomon to the white triangle ABC.



5. A rectangle has a 10 by 10 square gnomon. What are the dimensions of the rectangle?

6. We know that the first 10 Fibonacci numbers are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. Remember that we use the notation F_n to represent the nth Fibonacci number – that is, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc. Find the numerical value of the following:

   (a) F_{11}

   (b) $F_{11} + 2$

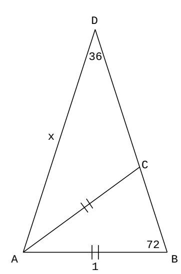
   (c) F_{11+2}

7. Given that $F_{36} = 14,930,352$ and $F_{37} = 24,157,817$, find:

   (a) F_{38}

   (b) F_{35}

8. *Golden Triangles:*



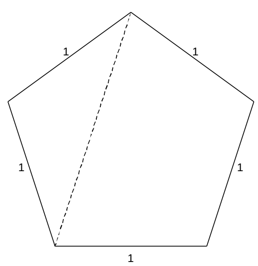
   (a) Show that triangle ABD is similar to triangle BCA.

(b) Use the similarity you showed in (a) to show that $x = \varphi = \dfrac{1+\sqrt{5}}{2}$.

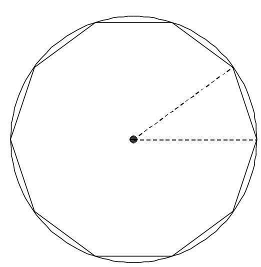
(c) Show that in the isosceles triangle ACD, the ratio of the longer side to the shorter side is again φ.

9. The regular pentagon in the following figure has sides of length 1. Show that the length of any one of its diagonals is φ.

   Hint: You may find it helpful to know that the angle a diagonal forms with the closest side of the pentagon is 36°.



10. *A relationship between φ and π:* A regular decagon (that is, a figure with 10 equal sides and 10 equal angles) can be inscribed in a circle of radius r, as shown below. Using $r = 1$, to make the calculations simpler,



(a) find the perimeter of the decagon in terms of φ, using the results of the Problem 8.

(b) use that the perimeter of the decagon and the circumference of the circle are roughly equal to find an approximate expression that relates φ and π.

11. Remember the amazing Binet's formula, which allows us to find F_N without having to first find the first $N-1$ Fibonacci numbers, and which accomplishes that by bringing φ in to the mix:

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}.$$

Verify that Binet's Formula works for $N = 3$. To be sure it works and that you're not introducing any round-off error, *do this without a calculator!*

12. In Binet's formula, we need to calculate powers of φ (and $\frac{1-\sqrt{5}}{2}$, which happens to be $-\frac{1}{\varphi}$). You probably found, in Problem 11, that this can be painful. In this problem, we see an easier way of calculating powers of φ, which makes it easier to understand how Binet's formula could feasibly be used without a calculator .

Remember that φ is one of two solutions to $x^2 - x - 1 = 0$ ($\frac{1-\sqrt{5}}{2}$ is the other). So $\varphi^2 - \varphi - 1 = 0$, or $\varphi^2 = \varphi + 1$.

(a) Show that $\varphi^3 = 2\varphi + 1$.

(b) Show that $\varphi^4 = 3\varphi + 2$.

(c) Show that $\varphi^5 = 5\varphi + 3$.

(d) Look at the results for φ^2, φ^3, φ^4, and φ^5. Based on what you see, what do you think φ^6 is ? Check your results.

(e) In general, how do you think φ^N can be rewritten, in terms of just a single power of φ and some whole numbers?

13. Below is a list of works of art often said to incorporate the Golden Ratio. Please pick one (or more), and photocopy it from a book. (Please do not use print-outs from the web, as they can be distorted in shape, and so your results will not mean much.)

- Giotto's *Madonna in Glory*

- Duccio's *Madonna Rucellai*
- Cimabue's *Santa Trinita Madonna*
- Dürer's *Adoration of the Magi*
- Da Vinci's *Mona Lisa*, *St. Jerome*, *Madonna on the Rocks* (either version), *A Head of an Old Man*, *Annunciation*
- Michelangelo's *David*
- Seurat's *The Bathers*
- Mondrian's *Place de la Concorde*
- Severini's *Maternity*

Once you've chosen your painting,

(a) Really look at it, and try to think of as many ways as possible that the Golden Ratio may have been used.

An artwork may incorporate the Golden Ratio in many ways. One obvious way would be if the painting itself were a Golden Rectangle, but there are lots of other possibilities:

- a line in the painting may be cut in the Mean and Extreme Ratio
- a rectangle in the painting may be a Golden Rectangle (that is, the ratio of long side to short side may be the Golden Ratio)
- an isosceles triangle in the painting may be a Golden Triangle (that is, the ratio of long side to short side may be the Golden Ratio)
- a rectangle fitting snugly around a figure in the painting may be a Golden Rectangle, and similarly for a Golden Triangle
- a body might have been drawn so that various parts are in the Golden Ratio
- the distances between several items may (when the ratio of those distances is taken) in the Golden Ratio, etc

Feel free, by the way, to read up on the claims relating to your choices on the web or in books (Mario Livio's chapter on "Painters and Poets have Equal License" discusses many but not all of the above). Just make sure you do the measuring yourself. Draw any lines as thinly as you can, as you're dealing with a much smaller version than the original, so a thin line on a shrunk version would correspond to a very thick line on the original.

(b) Measure all the distances you thought of in the previous part. Clearly label your measurements, and (as indicated in the previous part) draw the lines on the photocopy you're using.

(c) Decide upon an accuracy range for your measurements.

(d) Calculate all the ratios you thought of in the first part, including acceptance ranges for each.

(e) Does the Golden Ratio fall into any of these?

(f) Do you think the artist had the Golden Ratio in mind when creating this artwork? (This may not follow immediately from your previous results if, for instance, you think the Golden Ratio was close enough that it might have been intentional, even if it didn't fall into your ranges. After all, your ranges only reflect your measurement range, not any adjustments or errors the artist might have made.)