1. We know that the first 10 Fibonacci numbers are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. Remember that we use the notation F_n to represent the nth Fibonacci number – that is, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc. Find the numerical value of the following:

   (a) F_{11}

   (b) $F_{11} + 2$

   (c) F_{11+2}

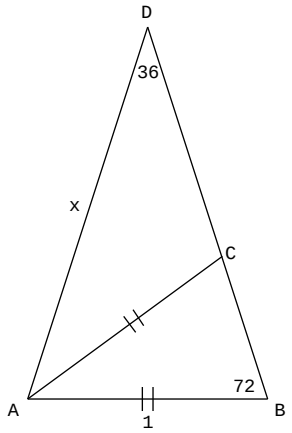
2. Given that $F_{36} = 14,930,352$ and $F_{37} = 24,157,817$, find:

   (a) F_{38}

   (b) F_{35}

3. Let a represent the 1000th Fibonacci number and b represent the 1001st Fibonacci number. Express the 1003rd Fibonacci number in terms of a and b. Simplify your answer.

4. *Golden Triangles:*



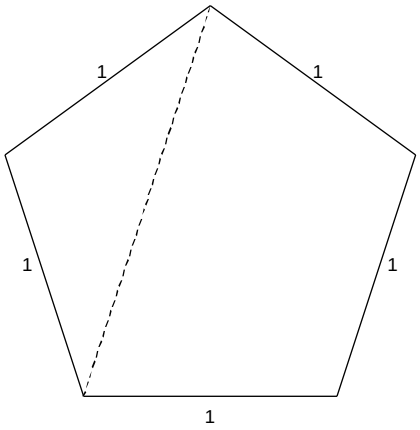
(a) Show that triangle ABD is similar to triangle BCA.
Hint: Remember that the base angles of an isosceles triangle are equal! Also remember that the sum of the angles in a triangle are $180°$.

(b) Use the similarity you showed in (a) to show that $x = \varphi = \dfrac{1+\sqrt{5}}{2}$.
Hint: Can you say anything about the length $\overline{CD}$?

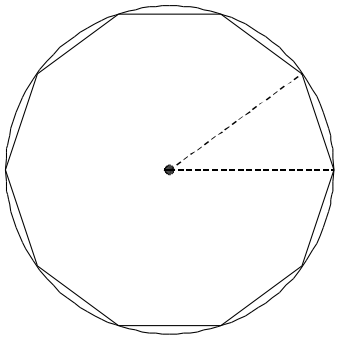
(c) Show that in the isosceles triangle ACD, the ratio of the longer side to the shorter side is again φ.

5. The regular pentagon in the following figure has sides of length 1. Show that the length of any one of its diagonals is φ.

   Hint: The angle a diagonal forms with the closest side of the pentagon is 36°. You may find it handy to use the results of Problem 4.



6. *A relationship between φ and π:* A regular decagon (that is, a figure with 10 equal sides and 10 equal angles) can be inscribed in a circle of radius r, as shown below. Using $r = 1$, to make the calculations simpler,



(a) find the perimeter of the decagon in terms of φ, using the results of the Problem 4.
Hint: Since a full circle is 360°, can you figure out what the angle I've shown at the center is?

(b) use that the perimeter of the decagon and the circumference of the circle are roughly equal to find an approximate expression that relates φ and π.

7. (This is an adaptation of Problem 2 from Lesson 1 of *Lessons in Mathematics and Art*; if you'd like to see the picture below in color, this text is available on the web at
`http://mypage.iu.edu/~mathart/viewpoints/lessons/` .)

   Below is a detail from Edgar Degas' painting, *The Rehearsal*, with picture plane coordinate axes superimposed. Using the points $M(216, 88)$, $N(249, 68)$, $P(283, 302)$, and $Q(317, 293)$ in the figure (the coordinates are in pixels), find:

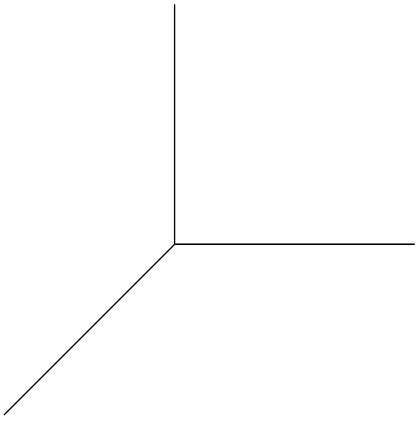
   (a) $d(M, N)$

   (b) $d(P, Q)$

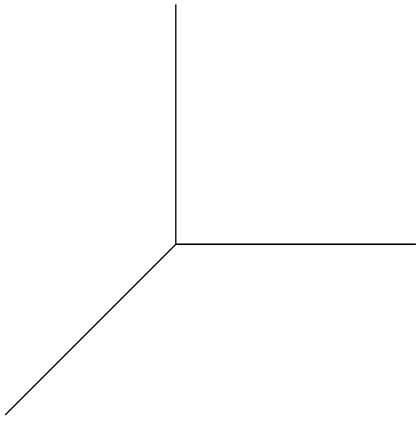
   (c) We've seen that in several systems of proportions, a person's foot should be about the same length as their forearm. In 3-space, the dancer's left foot and left forearm would be roughly parallel and directly above one another. We'll see later that because of that, if they were indeed the same length, then their images in the painting would also be the same length. Are they?

8. Please plot the following points on a set of 3-D coordinate axes. Mark units on your axes, and show enough dashed lines so I can see how you found where to put your points.

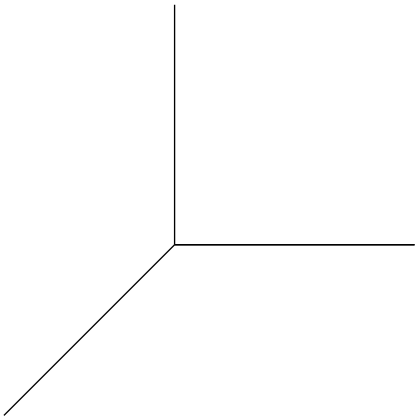
   (a) $A(1, 3, 4)$

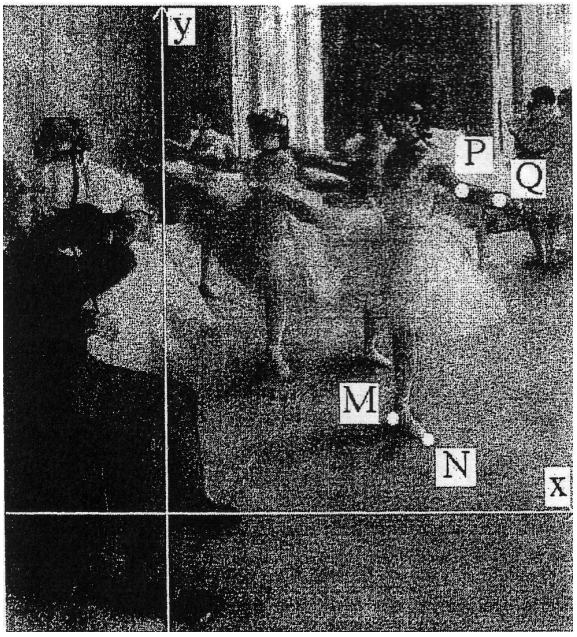
   

   (b) $B(2, 4, 0)$

   (c) $C(0, 3, -1)$


y
P
Q
M
N
x