

Exam 1 will cover through Problem Set 4 (gnomons, gnomonic growth, and golden rectangles).

- ADVICE:

- MOST IMPORTANT PIECE OF ADVICE: Please don't take the following words of advice as the final word on how to study – everybody learns differently. I would hate for someone to follow my advice to the letter if they can tell it's not going to work for them.
- Spread studying for this exam out over several days. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; *and* because each day you're stopping when tired or frustrated, it's actually a *more* efficient use of your time, and so leaves you more time for your other classes.
- In an ideal world, the best way to study for a math test is to re-read all the readings and your notes, summarize the topics we've covered, and re-do as many homework problems as possible.
If you are not living in an ideal world (and who is?), I would still skim the readings and the notes, but if you have to choose one thing to focus on, it should be to *do* (not to read) as great a variety of problems as possible. In addition to doing the few problems I've included on this study guide, you'll also want to *redo* as many problems as you can from the problem sets.
- While getting the solutions to the problem sets from the library can be helpful, reading through them is *not* enough for most people.
- When you're doing problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.
- When doing a problem that you've done before, don't waste your time trying to remember how you did it the first time—often, memory proves to be false and can lead you astray. Just focus on doing what makes sense.
- Should you study alone or with other people? That varies from person to person, but in general I'd say most of your studying should be on your own, particularly as it gets closer to the day of the exam. I think group study is best for most people at the beginning of the study process. Since the exam is individual, at some point in your studying, you have to be doing problems individually.
- How long should you study for this? A lot. "A lot" will vary from person to person also, but I'd suggest an absolute minimum of 6 hours. If you've struggled with the problem sets, then allow more time. If you breezed through the problem sets, then you *may* be able to get away with less studying— but why risk it?!

- TOPICS:

- Know the definitions of ratio and proportion, and how to use them.
- Review what a system of proportions is, who uses them, and why.
- Understand the Vitruvian system for human proportions, the Modulor, and the Sacred Cut. We spent the most time in class on the Sacred Cut, of course.
- Be able to describe a couple of different ways ancient Romans seem to have used the Sacred Cut in architecture.
- Acceptance Intervals, or Acceptance Ranges, for judging whether a ratio actually appears in art or architecture – where do they come from, how do we use them?
- Know the definition of the Extreme and Mean Ratio, and how it leads to the Golden Ratio.
- Judging whether a line is cut into Mean and Extreme ratio; figuring out how long to make a line if you want it to be cut into Mean and Extreme ratio and you know you want the short (or long, or whole) portion to be a certain length.
- How/where does the Golden Ratio shows up in a pentagon/pentagram?
- Understand how the golden ratio Φ and π both seem to appear in the Great Pyramid.
- Be able to work with similar figures
- Understand gnomons and be able to work with them.
- Know how the Golden Ratio can be derived from the notion of gnomons.
- What a Golden Rectangle is, what it has to do with φ , and what it has to do with gnomons, and how to construct one.

- PROBLEMS:

The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

A word of caution: You are responsible for all material covered in your reading, whether or not we covered it in class.

1. Use the Vitruvian system of proportion to decide how big you should make the distance from the middle of the breast to the crown if you've made the distance from the line between the eyes to the hairline 2" long.
2. Use geometry to find the length of the side of the Sacred Cut square, if the side of the outer square has length 2 .
3. You are designing a web page with two pictures that you would like to place side by side. For aesthetic reasons, you'd like them to be the same height, and yet the way the pictures are currently stored, one is taller than the other.

Here are the dimensions as they are stored on your computer:

Picture	Width (pixels)	Height (pixels)
Canaletto's <i>Piazza San Marco Looking South-East</i>	958	800
Rafael's <i>School of Athens</i>	800	554

4. You need to make the taller picture be the same height as the shorter one, without changing its shape.
 - (a) Which picture do you need to adjust?
 - (b) How high will it be?
 - (c) Use proportion to decide how wide it should be.
5. Even after adjusting one picture, these pictures are way too wide. If you want the total width of the two pictures to be 800 pixels,
 - (a) How high do they need to be, so that you haven't distorted either picture? (Use the results of part (a), so you only have to find one height.)
 - (b) How wide does each of the two pictures need to be, so that you haven't distorted either's shape?
6. Suppose Goldie measures her height and the height of her belly button. She only has a ruler, so the measurements aren't especially accurate: she finds her height to be $5'6'' \pm 2''$ and the height of her belly button to be $3'3'' \pm 1.5''$.

Find the interval in which the ratio of her height to her belly button must lie. Does the Golden Ratio lie in this interval?

7. Suppose you know you've measured the height of an archway accurate to within 3% and the width of the archway accurate to within 1.5%. If you found the height to be 83" and the width to be 56",

(a) What is the acceptance range for the ratio of height to width?

(b) Is $\sqrt{2}$ in this acceptance range?

8. For each of the following, you'll be drawing a line that is cut in mean and extreme ratio (i.e. the Golden Ratio).

(a) Suppose we want to draw a line cut in mean and extreme ratio, and we want the longer segment to have length 3. How long should the shorter segment be? Draw such a line as carefully as possible.

(b) Suppose we want to draw a line cut in mean and extreme ratio, and we want the short segment to be 5 units long. How long would the whole line be? Draw such a line as carefully as possible.

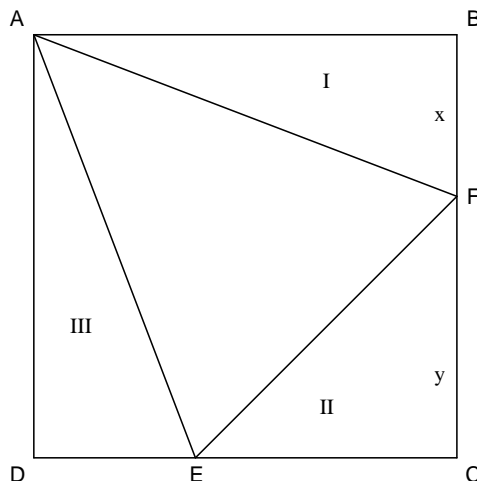
9. Suppose a splinter group in the class neither agrees with Euclid nor with the Very Cool ratio advocates. They feel strongly that the most beautiful way of cutting a line is as follows:

*A line is said to be cut in an **absolutely fabulous** ratio when the greater segment is to the lesser segment as the whole segment is to twice the greater.*

(a) What *is* this absolutely fabulous ratio? (That is, find what number it equals)

(b) Discuss the issue of the short segment versus the long segment.

10. In the following figure, $ABCD$ is a square, and the three triangles I , II , and III have equal areas. By following the steps below, you are going to show that $\frac{y}{x} = \varphi$.



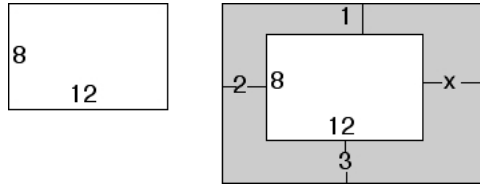
- (a) Find the area of triangle I, in terms of x and y .
- (b) Using that the areas of triangle I and triangle III are equal, find the length of \overline{DE} .
- (c) Find the length of \overline{EC} .
- (d) Find the area of triangle II.
- (e) Using that the areas of triangle I and triangle II are equal, show that

$$\left(\frac{y}{x}\right)^2 - \frac{y}{x} - 1 = 0.$$

- (f) Use the quadratic formula to find $\frac{y}{x}$.
11. You showed in your homework that a pyramid created by measuring the base using some number of revolutions of the drum, and the height by using twice that number of diameters of the drum is very close to being similar to the Great Pyramid at Gizeh.

While the Great Pyramid is the most famous of the pyramids, there are others at Gizeh (as well as throughout Egypt). The dimensions for the Second Pyramid at Gizeh are 470.75 feet high, with the sides of the base each being approximately 702' long. Is this pyramid also close to being similar to a pyramid that is constructed using the above technique?

12. Find the value of x so that the shaded "rectangular ring" is a gnomon to the white rectangle.



13. Rectangle A is 2 by 3. Rectangle B is a gnomon to rectangle A . What are the dimensions of rectangle B ?
14. A rectangle has a square gnomon. The new rectangle obtained by attaching the square gnomon to the original rectangle has longer leg 20. What are the dimensions of the original rectangle?