- WARNING: Please don't take this as the final word on how to study. First of all, everybody learns differently, second of all, I am an expert at math, not at the theory of studying, and finally, I'm squeezing this in among all the *other* things I have to get done (much as you're doing with all of your work), and so I may not think of everything. I would hate for someone to follow my advice to the letter (perhaps against their better judgment) and have it not work for them.

- ADVICE:

  - Let me emphasize this again –I know you have other classes, but spread studying for this exam out over several days. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc

  - In an ideal world, the best way to study for a math test is to re-read all the readings (including your notes – this course is definitely heavily notes-based!), summarize the topics we've covered, and re-do as many homework problems as possible.

    If you are not living in an ideal world (and who is), I would still skim the readings, and in the notes from class try to emphasize connections with math and art that may not have been covered much in the readings. Your main focus, however, should be to *do* (not to read) as great a variety of problems as possible. In addition to doing the few problems I've included on this study guide, you'll also want to redo as many problems as you can from the first three problem sets. (Notice that I said "redo" – simply reading through solutions doesn't do it.)

  - When you're doing problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.

  - When doing a problem that you've done before, don't waste your time trying to remember how you did before–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense.

  - Should you study alone or with other people? That varies from person to person, but in general I'd say most of your studying should be on your own, particularly as it gets closer to the day of the exam. I think group study is best for most people at the beginning of the study process. Since the exam is individual, at some point in your studying, you have to be doing problems individually.

  - How long should you study for this? Alot. "Alot" will vary from person to person also, but I'd suggest an absolute minimum of 6 hours. If you've struggled with the problem sets, then leave more. If you breezed through the problem sets, then you *may* be able to get away with less – but why risk it?!

- Topics:
  - What a Golden Triangle is, what it has to do with φ and what it has to do with gnomons
  - Fibonacci numbers
  - How the Fibonacci numbers are related to φ
    - sequence of $\frac{F_n}{F_{n-1}}$
    - Binet's formula
    - anything else you can think of
  - Using Binet's formula
  - How/where the Golden Ratio shows up in a pentagon/pentagram.
  - The distance formulae for points in 2-space and for points in 3-space
  - Plotting points in 3-space
  - The relationship between points in 3-space (as in all our cube problems)
  - The Perspective Theorem-where it comes from, and using it
  - The meaning of the word "orthogonal"
  - Vanishing points - where do images of lines orthogonal to the picture plane vanish? How about lines parallel to the picture plane (the xy-plane)? Lines parallel to the "floor" (the xz-plane)? Lines parallel to a "side wall" (the yz-plane)?
  - Vanishing points of parallel lines
  - Finding the correct viewing position for a drawing in one-point perspective.
  - The rules of perspective (remember, I gave you a handout with several such rules)
  - Subdividing rectangles into halves, fourths, eighths.
  - Duplicating a rectangle immediately next to (attached to) your original.

- Problems:

  The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

  A word of caution: You are responsible for all material covered in your reading, whether or not we covered it in class.

  1. *Inspired by Number 6 from* Excursions in Modern Math*, Chapter 9*: Use that $F_{26} = 121,393$ and that $F_{28} = 317,811$, to find F_{27}.
  2. *Also inspired by Number 6 from* Excursions in Modern Math*, Chapter 9*: Use that $F_{21} = 10,946$ and that $F_{23} = 28,657$,to find F_{20}.

3. Let a represent the 300th Fibonacci number and b represent the 301st Fibonacci number. Express the 298th Fibonacci number in terms of a and b. Simplify your answer.

4. *Inspired by Number 13 from* Excursions in Modern Math*, Chapter 9*: Fact: $(F_1 + F_2 + F_3 + \ldots + F_N) + 1 = F_{N+2}$. Verify this fact for:

   (a) $N = 4$

   (b) $N = 10$

   Hint: Verify means show that the statement is true when $N = 4$, or $N = 10$. That is, when (for instance) $N = 4$, find the left side and show that it's the same as the right side.

5. Remember the amazing Binet's formula, which allows us to find F_N without having to first find the first $N - 1$ Fibonacci numbers, and which accomplishes that by bringing φ in to the mix:

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}.$$

   Verify that Binet's Formula works for $N = 3$. To be sure it works and that you're not introducing any round-off error, *do this without a calculator!*

6. *Calculating powers of φ.*

   Remember that φ is one of two solutions to $x^2 - x - 1 = 0$ ($\frac{1-\sqrt{5}}{2}$ is the other). So $\varphi^2 - \varphi - 1 = 0$, or $\varphi^2 = \varphi + 1$.

   (a) Use that $\varphi^3 = \varphi^2 \cdot \varphi$, along with the above relationship, to show that $\varphi^3 = 2\varphi + 1$.

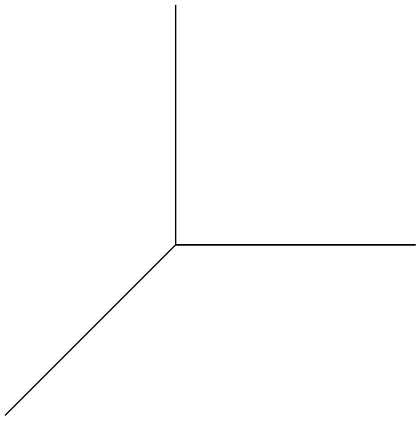
   (b) Use your result for φ^3 to show that $\varphi^4 = 3\varphi + 2$.

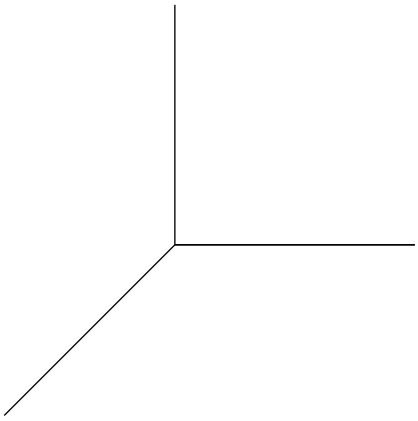
   (c) Show that $\varphi^5 = 5\varphi + 3$.

   (d) Look at the results for φ^2, φ^3, φ^4, and φ^5. Based on what you see, what do you think φ^6 is ? Check your results.

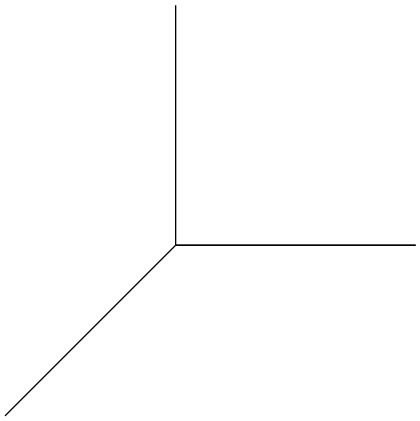
   (e) In general, how do you think φ^N can be rewritten, in terms of just a single power of φ and some whole numbers?

7. Please plot the following points on a set of 3-D coordinate axes.
   (a) $A(2, 0, -3)$ (b) $B(3, 1, 2)$





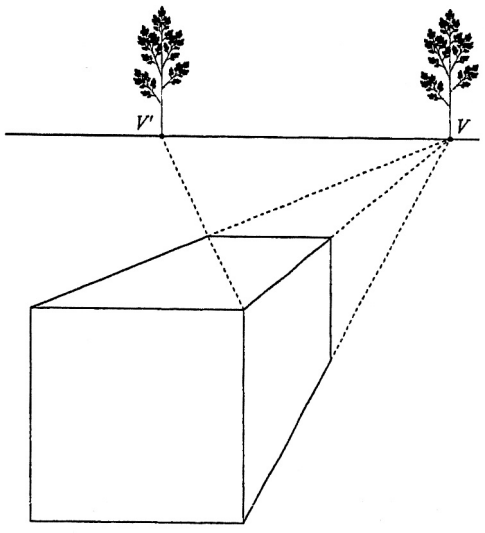
   (c) $C(-3, -1, 2)$



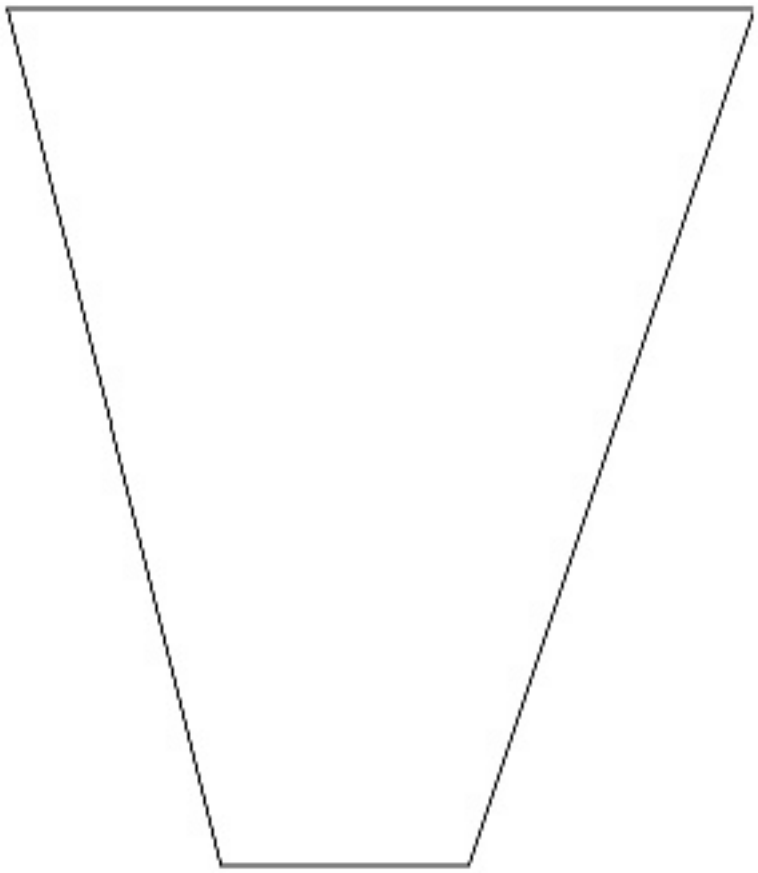
8. Consider a box whose faces are parallel to the coordinate planes. Suppose the coordinates of two opposing corners of the box have coordinates $(-7, -10, 5)$ and $(-3, -4, 13)$.
   (a) How wide is the box in the x direction?
   (b) How tall is the box in the y direction?
   (c) How deep is the box in the z direction?
   (d) What are the coordinates of the remaining 6 corners?
   (e) Use the Perspective Theorem to find the coordinates of the perspective image of each of the eight corners. Use a viewing distance of 10 units.
   (f) Plot the images you found in the previous part in the xy plane. Be careful,

and use graph paper . Use straight lines to connect the corners that are connected on the real box (of course, use dashed lines for the hidden edges.)

9. Suppose this time we have a cube whose faces are again parallel to the coordinate planes, but only the coordinates of one corner are known. Suppose the bottom left back corner has coordinates $(1, -3, 2)$ and that the length of each edge is 7.
   (a) What are the coordinates of the other seven corners of the cube?
   (b) Use the Perspective Theorem to find the perspective image of each of the eight corners. Use a viewing distance of 2 units.
   (c) Plot the images you found in the part in the xy plane, on graph paper.
10. For this problem, you will need to print out from the web (or photocopy from a book) a picture of Piero della Francesca's *The Flagellation*. Here is one website that has it:
    `http://www.wga.hu/frames-e.html?/html/p/piero/francesc/flagella.html`
    (If you click on the image, you can see a larger version.)
    (a) Locate the primary vanishing point.
    (b) If there are any secondary vanishing points, find one.
    (c) Determine the correct viewing position.
11. If the box below represents a cube, then we can use our usual techniques to find the correct viewing position. But suppose the box is *not* a cube. Suppose instead that for whatever reason we know that the *side* of the box is intended to be twice as deep from left to right (that is, from front to back) as it is tall. What is the viewing distance in this case?



12. Divide the rectangle below in half. (Cut the lines which no longer appear parallel in half.) Then divide the nearer of your halves in half; the nearer of your quarters in half, and the nearer of your eighths in half. In the end, the rectangle should have one half, one fourth, one eighth and two sixteenths.



13. Beginning with the rectangle shown below, draw a portion of a brick wall 3 bricks wide and 4 bricks high. Remember that in order to do this so it really looks like a brick wall, the second row of bricks must be offset from the first row, so that the end of one brick divides the brick below it in half.