- WARNING: As usual, I am squeezed for time as I write this, so please do not assume that it is a stand-alone study resource.
- ADVICE:
  - If you want to look through the webpages I show sometimes in class, here's the address:
    `http://acunix.wheatonma.edu/jsklensk/Art_Spring07/Inclass.html`.
  - Part of what I hope to learn on a test is the extent to which you can take ideas we've learned and apply them to situations that are different from what you've seen before. So your goal is to learn not only how to do problems we've covered before, but to understand *why* we do them that way – in other words, to understand the ideas behind them. Of course, in some cases there are only so many questions to ask, and so I may not be able to come up with dramatically new and different questions, but if you understand the concepts and can apply them, you should be prepared for most anything that comes along.
  - I remind you once again to spread studying for this exam out over several days. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc
  - Review your notes and the readings. In the notes from class, remind yourself of the connections between math and art that may not have been covered much in the readings.
  - Your main focus should be to *do* (not to read) as great a variety of problems as possible. In addition to doing the few problems I've included on this study guide, you'll also want to redo as many problems as you can from the first three problem sets. (Notice again that I said "redo" – simply reading through solutions doesn't do it.)
  - It's important enough that I'll mention it again – When you're doing problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.
  - When doing a problem that you've done before, don't waste your time trying to remember how you did before–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense. The whole joy of math is the logical path your thoughts make – when you look back upon a solution, it should look like an inexorable journey, where no other choices really made sense.
  - Should you study alone or with other people? That varies from person to person, but in general I'd say most of your studying should be on your own, particularly as it gets closer to the day of the exam. I think group study is best for most people at the beginning of the study process. Since the exam is individual, at some point in your studying, you have to be doing problems individually.

- How long should you study for this? Alot. "Alot" will vary from person to person also, but I'd suggest an absolute minimum of 6 hours. I know one person in your class studied 20 hours for the last test – and it worked! If you've struggled with the problem sets, then leave more. If you breezed through the problem sets on your own, then you *may* be able to get away with less – but why risk it?!

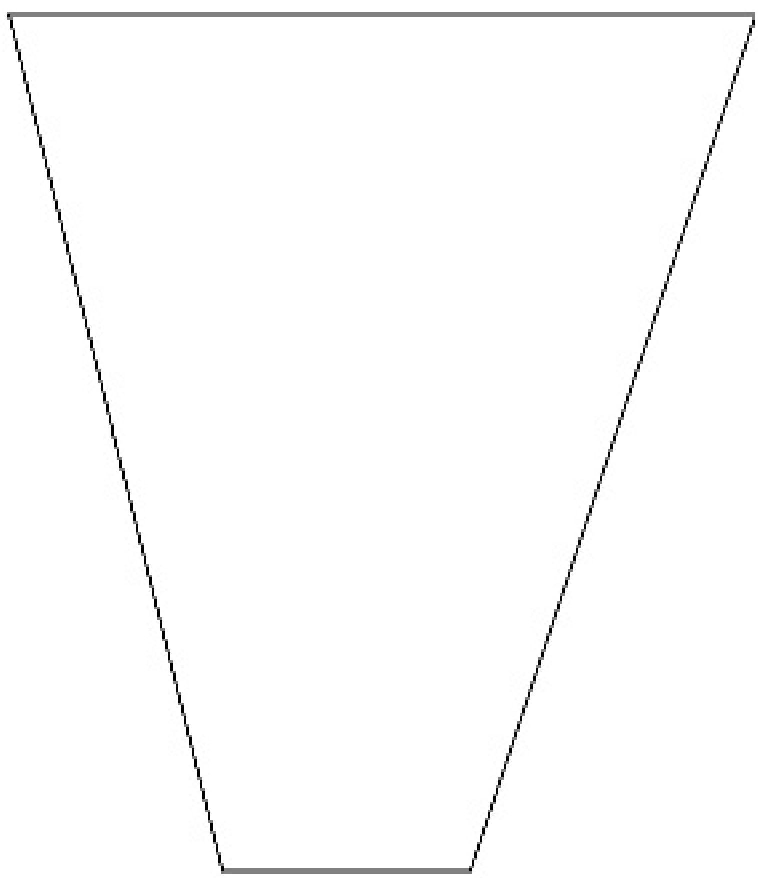
- TOPICS:
  - Subdividing rectangles into portions that are *not* powers of 2 – thirds, fifths, etc. (This is from homework)
  - Duplicating a rectangle so that there's an arbitrary amount of separation between the two, including the possibility of overlap. (This is also from homework).
  - Anamorphic art – drawing a picture that appears distorted unless viewed from an extreme viewpoint. We focused on planar anamorphic art. Be sure you understand the ideas behind it, as well as how to *do* it. I will not be including any new problems on this in this study guide, but that doesn't mean you don't need to know it. Be sure you know which grid you draw the undistorted picture on, and which grid you draw the final distorted version on.
  - The connection between the 4th dimension, non-Euclidean geometry, and early 20th century art – who were some of the artists that we know were influenced by these mathematical notions? (Juan Gris, Jean Metzinger, Albert Gleizes, Marcel DuChamp, and Max Weber are the ones whose work we saw in class.)
  - What a Linelander would see when a 2 dimensional object passes through its space, what a Flatlander would see when a 3 dimensional object passes through its space, and what we would see when a 4 dimensional object passes through our space. (I would only ask this for the appropriately dimensioned analog of the cube or sphere)
  - Looking for patterns in the number of vertices, edges, faces, solids, etc in a point, line segment, square and cube, and using the patterns to predict how many vertices, edges, faces, solids, 4-d regions and 5-d regions are in the hypercube, hyperhypercube, hypertetrahedron, and hyperhypertetrahedron.
  - Ways to represent the hypercube in 2 and 3 dimensions.
  - Which of Euclid's five postulates are we discarding in non-Euclidean geometry? Be able to state this postulate. (It's on one of the webpages available through my unlinked website.)
  - Describe a model of elliptic space, and what lines and circles look like on this model.
  - Discuss triangles in elliptic space (what they look like, the sum of their angles, and their areas).
  - Be able to describe at least one model of hyperbolic space.

  - What do lines look like on the Poincaré disc model of hyperbolic space?
  - Name an artist who was interested in hyperbolic space and did several works of art relating to it. (Marcel Duchamp)

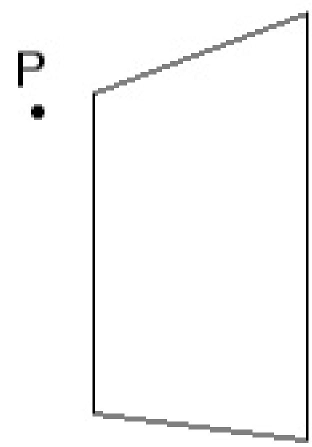
- PROBLEMS:

  The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

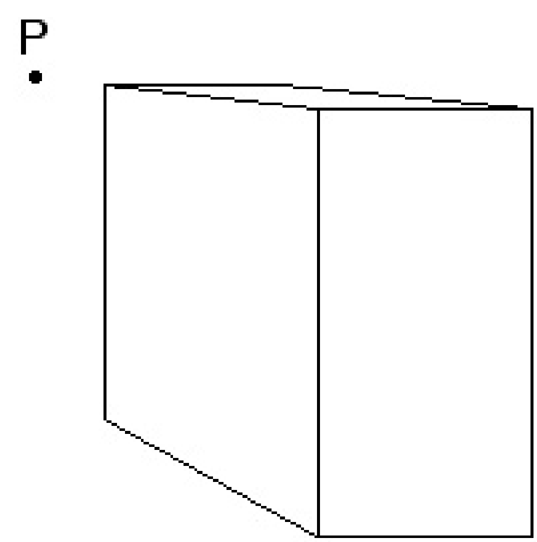
  1. On the perspective drawing of a rectangle below, draw a horizontal line cutting the sides which no longer appear parallel into the division one-ninth/eight-ninths. Probably the easiest way to do this, using the "stacking rectangle" technique we developed for the homework, is to divide the rectangle into thirds, and then one of the thirds on an end into thirds again.



2. Below is a perspective drawing of a window, retreating orthogonal to the picture plane. Draw a duplicate of this window, so that it's upper rear corner is located at the point P, to create the appearance of a partially open sliding glass door.



3. Below is a perspective drawing of a box, along with a point P. Draw a duplicate of the box, using the techniques we've developed. Place the duplicate so that it's front left corner (as we face it) is located at the point P.



4. (Exercise 7 from Chapter 6, Section 1: Other Dimensions, other worlds; Flatlands) A. Square has a conversation with his grandson.

   > 'I began to show the boy how a Point by moving through a length of three inches makes a Line of three inches, which may be represented by 3; and how a Line of three inches, moving parallel to itself through a length of three inches, makes a Square of three inches every way, which may be represented by 3^2 [square inches].'

   Generalize to give a geometric significance to the quantity 3^3, thus answering a question posed by the grandson.

5. What would A. Square observe (assuming he has the leisure to walk around and perhaps even touch), if a cube passed through Flatland:

   (a) with its bottom parallel to the plane of Flatland?

   (b) edge first? (this is trickier, but you can try it out for yourself – and we saw it in the movie on the hypercube)

   (c) with one corner approaching Flatland before the rest? (and this is the trickiest, but again, we saw it in the movie, and you can try to replicate it.)

6. What would a 4-dimensional cube look like to us if it passed through our 3-dimensional space "flush" with our space? (I'm of course having trouble finding words to describe, since we don't have many words to describe hyperspace, but what I'm asking you to do is the analogy of part (a) in the previous problem). Describe and draw pictures as necessary to make your point.

7. We have seen in class and in homework how we can determine the number of vertices v, edges e, faces f, solids s, 4-dimensional regions t and 5-dimensional regions u that are in a hypercube, a hyperhypercube, a hypertetrahedron, and a hyperhypertetrahedron. The results we found for both are summarized in the table below:

Dimension Figure	0D point	1D line segment	2D square	3D cube	4D hypercube	5D hyperhypercube
vertices v	1	2	4	8	16	32
edges e	0	1	4	12	32	80
faces f	0	0	1	6	24	80
solids s	0	0	0	1	8	40
4D regions t	0	0	0	0	1	10
5D regions u	0	0	0	0	0	1

We can define something called the *Euler characteristic* of any figure in any dimension. For figures in five dimensions or less, the Euler characteristic is defined as follows:

$$\text{Euler characteristic} = \chi = v - e + f - s + t - u.$$

Find the Euler characteristic χ of each of the figures listed in the above table. Notice anything?

8. What is the most likely intersection of a line and a 3-space in 4-space? You can either use the formula you developed in the most recent problem set, or you can think analogously to situations you can picture.

9. What does a line on the sphere (one of our models of elliptic space) look like? Is there a unique line through *any* two points on the sphere?

10. Does the concept of parallel lines exist in elliptic space? If so, can there be two lines through the same point that are *both* parallel to a third line not through the point? (This is impossible in Euclidean space).

11. Is there a unique line through *any* two points in hyperbolic space?

12. Does the concept of parallel lines exist in hyperbolic space? If so, can there be two lines through the same point that are *both* parallel to a third line not through the point?

13. What can you say about the area of a circle in elliptic space? In hyperbolic space?

    (a) Find a triangle on the surface of the sphere whose angles add up to 270°. Is the area of this triangle equal to, greater than, or less than

    $$\frac{1}{2} \times \text{base} \times \text{height?}$$

    (b) Can you find a triangle on the sphere whose angles add up to less than 180°?