Most of these exercises are either from Chapter 6 in Symmetry, Shape, and Space*, or are inspired by that chapter.*

1. In *Flatland*, A. Square has a conversation with his grandson, reproduced below:

   > 'I began to show the boy how a Point by moving through a length of three inches makes a Line of three inches, which may be represented by 3; and how a Line of three inches, moving parallel to itself through a length of three inches, makes a Square of three inches every way, which may be represented by 3^2 [square inches].'

   Generalize to give a geometric significance to the quantity 3^3, thus answering a question posed by the grandson.

2. What would A. Square observe (assuming he has both the time and the effrontery to walk around and perhaps even touch), if a cube passed through Flatland:

   (a) with its bottom parallel to the plane of Flatland?

   (b) with one corner approaching Flatland before the rest? (This one is tricky – do the best with it you can!)

3. What would a 4-dimensional cube look like to us if it passed through our 3-dimensional space "flush" with our space? (I'm of course having trouble finding words to describe, since we don't have many words to describe hyperspace, but what I'm asking you to do is the analogy of part (a) in the previous problem). Describe and draw pictures as necessary to make your point.

4. A circle is the set of all points in a plane equidistant from the center. A sphere is the set of all points in space equidistant from the center.

   (a) How would you define a fourth dimensional sphere, called a hypersphere?

   (b) What would we see if a hypersphere passed through our space? Determine this by thinking analogously – what did A. Square observe as the sphere passed through his space?

5. Define a **hyperhypercube** in the 5th dimension analogously to how we defined the hypercube in the 4th:

   > Begin with a hypercube. Place a copy of it parallel to the original in the fifth dimension in "just the right position" (as we do when forming squares and cubes). Attach each vertex of the original hypercube to the corresponding vertex of the copy.

   Using our discussion of the hypercube as inspiration,

   (a) How many vertices does the hyperhypercube have, and why? (When answering the why part of the question, discuss this in the context of how the hyperhypercube is formed, rather than simply referring to patterns. Feel free to refer to our work on the hypercube.)

   (b) How many edges does the hyperhypercube have, and why? (Again, discuss in the context of how its formed.)

   (c) How many 2-dimensional faces does the hyperhypercube have, and why? (Same type of discussion as above.)

(d) How many 3-dimensional solids does it have? Justify.

(e) How many 4-dimensional regions? Justify.

(f) How many 5-dimensional regions? Justify.