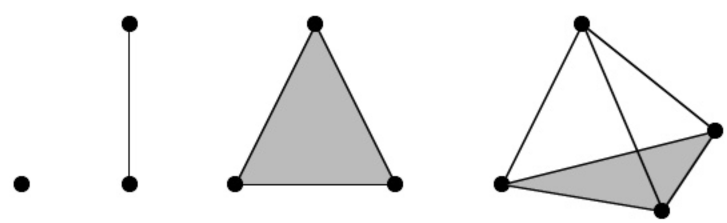
These problems are again all either from Symmetry, Shape and Space*, chapter 6: "Other Dimensions, Other Worlds",* adaptations of problems from that chapter, or inspired by that chapter.

1. (Adaptation of Exercise 14, §6.2) Cubes and hypercubes are higher-dimensional analogies of the two-dimensional square. On page 203-204, your text discusses how another type of higher-dimensional figure can be built up in analogy with the triangle:

   > *Start with a single point. In the next generation, add another point above the first, and connect the two, creating a line segment. Place that line segment on the floor and add another point above the line segment and connect this point with each of the points on the line segment, obtaining a triangle. Place the triangle flat on the floor and add another point above it; connecting the new point to each of the points in the triangle gives a three-dimensional solid called the tetrahedron or triangular pyramid.*

   > *Adding another point in the fourth dimension,* ana *or* kata *the figure, and connecting this point to each point in the tetrahedron gives a figure called the* hypertetrahedron *or* pentahedroid.

   Develop analogous formulae to those we developed for the hypercube to fill in the table You will use much the same thought process but of course since the figures you're working with are different, the formulas you develop will also be different .

Dimension	0D	1D	2D	3D	4D
Figure	point	segment	triangle	tetrahedron	hypertetrahedron
vertices v					
edges e					
faces f					
solids s					
4D regions t					

2. If we can ponder a 4th spatial dimension, we can also ponder a 5th. In 5 dimensional space, there will be 5 different fundamental directions of movement: up/down, left/right, in/out, ana/kata plus the additional 5th dimensional directions, for which I do not know a name.

   Define a **hyperhypercube** in the 5th dimension analogously to how we defined the hypercube in the 4th:

   > Begin with a hypercube. Place a copy of it parallel to the original in the fifth dimension in "just the right position" (as we do when forming squares and cubes). Attach each vertex of the original hypercube to the corresponding vertex of the copy.

   Throughout the following questions on the hyperhypercube, please feel free to refer to our work on the hypercube, either in your notes or on the relevant web page, or both. Each of the following questions is really two questions: a"how many" question, and a "why" question. Although you're welcome to use a formula to answer the "how many" portion of each question, when you answer the "why" portion, please discuss it in the context of how the hyperhypercube is formed, rather than simply referring to patterns in the numbers or to the existence of the formulas. In other words, what sort of thought processes did we use to develop the formulas?

(a) How many vertices does the hyperhypercube have, and why? ()

(b) How many edges does the hyperhypercube have, and why?

(c) How many 2-dimensional faces does the hyperhypercube have, and why?

(d) How many 3-dimensional solids does it have?

(e) How many 4-dimensional regions?

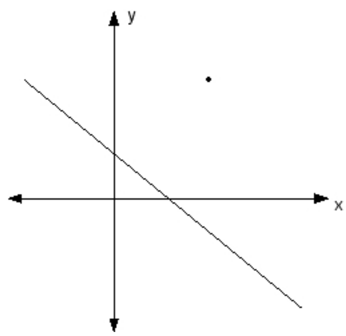
(f) How many 5-dimensional regions?

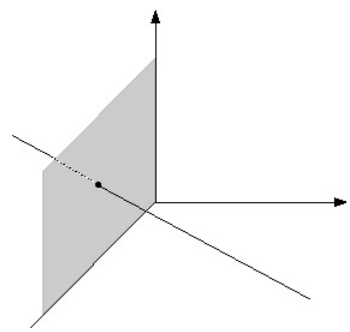
3. (Exercise 3, §6.2) In the plane, note that any two non-parallel lines intersect in a point.

   (a) Generalize this idea to the intersection of two non-parallel planes in the third dimension. That is, what does the intersection of any two non-parallel planes look like? Use sheets of paper (or an equivalent) as planes to help you figure this out.

   (b) Generalize even further, this time to the intersection of two non-parallel three-dimensional spaces in hyperspace. For this, of course, you'll need to use analogy to the dimensions you *can* picture. In other words, look for patterns in the two lower dimensions we've discussed so far.

4. (Exercise 4, §6.2) In the illustration on the left below, you see that "in general", (that is, usually) a point and a line on the plane will not intersect at all. In other words, given a randomly chosen line and a randomly chosen point in the same plane, more likely than not the point will not be on the line. In the illustration shown on the right, you see that "in general", a line and a plane in 3 dimensional space will intersect in a point.





(Of course, this does not always occur: the line could be parallel to the plane, or the line could lie on the plane, but given an randomly chosen line in space and a randomly chosen plane in space, *more often than not* the line will intersect the plane in only one point.)

(a) What is the most likely intersection of two lines in the plane? *Draw a sketch to illustrate.*

(b) What is the most likely intersection of two lines in 3-space? *Draw a sketch to illustrate.*

(c) Look at the results for this exercise, as well as those of part (a) of the previous exercise. In each case, we were looking at the intersection of either a point (0D), a line (1D), or plane (2D) with a line (1D) or plane (2D). Furthermore, in each case, the intersection occurred in either the plane (2D) or space (3D). And in each case, the resulting intersection was either nothing (no dimension), a point (0D), or a line (1D). In Part (b) of the previous exercise, you generalized the intersection of two 3D space in hyperspace.

Use your results to fill in the table on the next page that compare the dimension of the two intersecting objects, the dimension of the space the intersection occurs in, and the dimension of the most likely resulting intersection. (I've filled in the two facts that I gave you.)

Description of intersection	Dimension of object 1	Dimension of object 2	Dimension of space in which intersection occurs	Dimension of object formed by the intersection
Point with line in the plane	0	1	2	no intersection, so none (**not** 0)
Line with line in the plane				
Line with line in space				
Line with plane in space	1	2	3	point, so 0
Plane with plane in space				
Space with space in hyperspace (4D)				

(d) Scrutinize and ponder your completed table. Look for a relationship between the dimension of object 1, the dimension of object 2, the dimension of the space they sit in, and the dimension of the resulting intersection. Turn that relationship into a formula that relates the dimension of the resulting intersection to the other three quantitities: the dimensions of the two intersecting objects and the dimension of the space.

(e) What is the most likely intersection of a line and a plane in hyperspace? Justify your answer using whatever balance of insight into the 4th dimension and the formula you developed in the previous part that works for you.

(f) What is the most likely intersection of two planes in hyperspace? Again, justify your answer

(g) What is the most likely intersection of a plane and a 3-space in hyperspace?