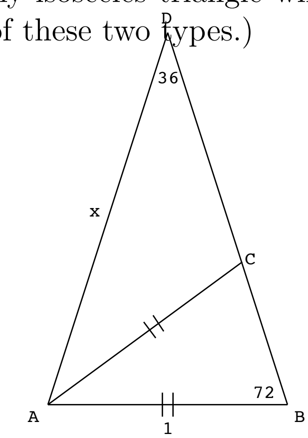
1. *Golden Triangles:* In this exercise, you will be showing that all of the triangles shown in the triangle below have the ratio *long side*:*short side* in the Golden Ratio. Since any triangles with the same angles will be similar, and hence have sides in the same proportions, this will ultimately show that any 72°-72°-36° triangle or any 36°-36°-108° triangle will be *Golden*, although you will of course first have to discover those angles. (Also, any isosceles triangle whose sides have this ratio will be similar to one of these two types.)



   (a) Show that triangle ABD is similar to triangle BCA.
   Hints:
   - Since you're given that $\overline{AB} = \overline{AC}$, you know that triangle BCA is an isosceles triangle.
   - Remember that the base angles of an isosceles triangle are equal, *and* if the base angles of a triangle are equal, it's an isosceles triangle.
   - Although triangle ABD *looks* like an isosceles triangle, you are not given that it is.
   - Also remember that the sum of the angles in a triangle are 180°.

(b) Show that triangle ADC is also an isosceles triangle.

(c) Use the result of part (b) to write the length $\overline{BC}$ in terms of x.

(d) Use the similarity you showed in (a), and your result to part (c), to show that $x = \varphi = \dfrac{1+\sqrt{5}}{2}$.

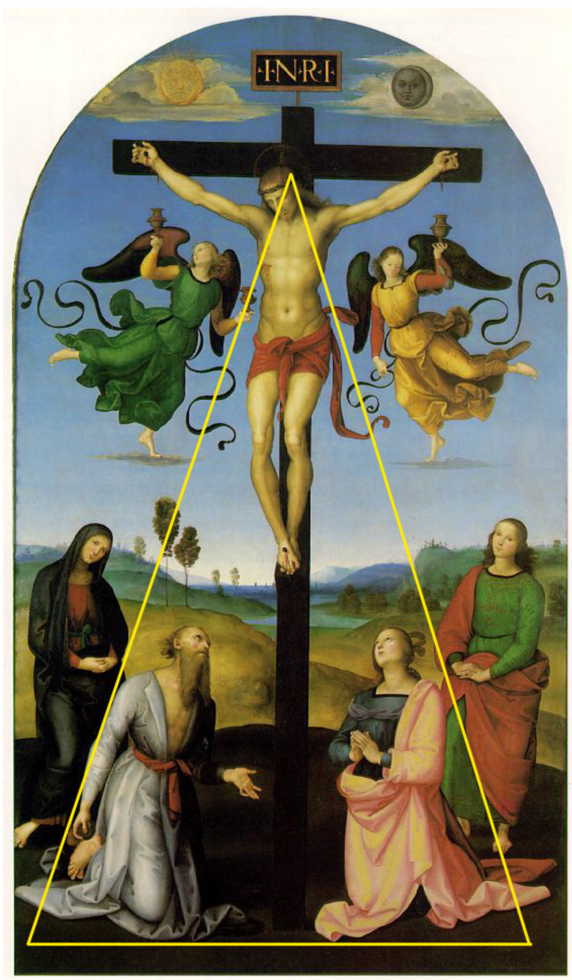
(e) Show that in the isosceles triangle ADC, the ratio of the longer side to the shorter side is again φ.

2. I found a couple of websites that claim that Raphael's *Mond Crucifixion* exhibits a Golden Triangle. Below is a replica of the diagram I found on the web.

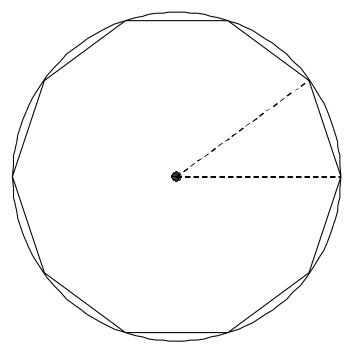
   (a) By carefully measuring this diagram, find the ratio of the length of the long side of this triangle to the base.

   (b) Assuming a 2% margin of error in your measurements, does this triangle fall within the margin of error being a Golden Triangle?

(c) Do you think the placement of this triangle is defined in a natural or obvious way by the features of the painting, or do you think it was conveniently chosen to create a nearly golden triangle? (For color and better detail, see problem set online). In case the lines obscure some of the detail: the top vertex is at the point where the horizontal line defined by the bottom of the crossbar intersects the midline defined by Jesus' body; this point is also the rightmost edge of the crown of thorns. The left side of the triangle extends to follow the arm of St. Jerome (kneeling), while the right side extends to follow the cloak of John the Evangelist (standing). The horizontal leg just grazes the robe of St. Jerome on the left and the cloak of Mary Magdalene on the right.



3. *Another relationship between φ and π:* A regular decagon (that is, a figure with 10 equal sides and 10 equal angles) can be inscribed in a circle of radius r, as shown below. Using $r = 1$, to make the calculations simpler,



   (a) find the perimeter of the decagon in terms of φ, using the results of Problem 1.
   Hint: You'll need to find the length of the sides of the decagon. Since a full circle is $360°$, can you figure out what the angle I've shown at the center is?

   (b) use that the perimeter of the decagon and the circumference of the circle are roughly equal to find an approximate expression that relates φ and π.

4. We know that the first 10 Fibonacci numbers are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. Remember that we use the notation F_n to represent the nth Fibonacci number – that is, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc. Find the numerical value of the following:

   (a) F_{11}

   (b) $F_{11} + 2$

   (c) F_{11+2}

5. Given that $F_{36} = 14,930,352$ and $F_{37} = 24,157,817$, find:

   (a) F_{38}

   (b) F_{35}

6. Let a represent the 1000th Fibonacci number and b represent the 1001st Fibonacci number. Express the 1003rd Fibonacci number in terms of a and b. (In other words, you're doing this without ever knowing what the 1000th and 1001st Fibonacci numbers are.) Simplify your answer.

7. Binet's Formula defines Fibonacci numbers directly:

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}.$$

Use Binet's Formula to find the 15th Fibonacci number.

Hint: Don't try to do this by hand - -use a calculator, and be very careful with your parentheses! If you do it all on the calculator (that is, without writing down rounded intermediate results and then re-entering those values later), you *should* end up with a whole number, with no rounding necessary, which always seems miraculous!