- WARNING: As usual, please do not assume that it is a stand-alone study resource.
- ADVICE:
  - The webpages I show sometimes in class have the address: `http://acunix.wheatonma.edu/jsklensk/Art_Spring09/Inclass.html`.
  - Part of the goal of any class is to stretch your ability to to take ideas we've studied and apply them to situations that are different from what you've seen before. Thus your goal when studying is not only to learn how to do problems we've done before, but to understand *why* we do them that way , so you understand the ideas behind them. If you understand the concepts and can apply them, you should be prepared for most anything that comes along.
  - As always, spread studying for this exam out over several days - it's better all around.
  - Review your notes and the readings. Remind yourself of the connections between math and art that may not have been covered much in the readings.
  - As always, your main focus should be to *do* as great a variety of problems as possible.
  - Give some thought to how different problems are connected, and why the steps within a problem make sense.
  - When doing a problem that you've done before, don't waste your time trying to remember how you did it the last time–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense. The whole joy of math is the logical path your thoughts make – when you look back upon a solution, it should look like an inexorable journey, where no other choices really made sense.
  - Whether to study individually or in a group is your choice; just remember that the exam is individual, so at some point in your studying, you should be doing problems individually.
- TOPICS:
  - I'm going to repeat two concepts, just to make sure they sunk in: subdividing a rectangle drawn in perspective into halves, quarters, eighths, etc, and drawing an attached copy of a rectangle drawn in perspective.
  - Subdividing rectangles into portions that are *not* powers of 2 – thirds, fifths, etc.
  - Duplicating a rectangle so that there's an arbitrary amount of separation between the two, including the possibility of overlap.
  - Anamorphic art – drawing a picture that appears distorted unless viewed from an extreme viewpoint. We focused on planar anamorphic art. Be sure you understand the ideas behind it, as well as how to *do* it. **I will not be including any**

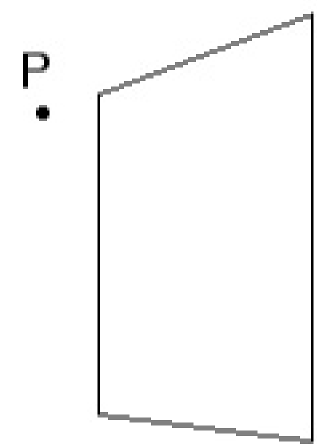
new problems on this in this study guide, but you still need to know how to do it. Be sure you know which grid you draw the undistorted picture on, and which grid you draw the final distorted version on.

- The connection between the 4th dimension, non-Euclidean geometry, and early 20th century art – who were some of the artists that we know were influenced by these mathematical notions? (Juan Gris, Jean Metzinger, Albert Gleizes, Marcel DuChamp, and Max Weber are the ones whose work we saw in class.)
- What a Linelander would see when a 2 dimensional object passes through its space, what a Flatlander would see when a 3 dimensional object passes through its space, and what we would see when a 4 dimensional object passes through our space. (I would only ask this for the appropriately dimensioned analog of the cube or sphere). By *see*, I really mean *experience through a combination of touch and sight, if given enough time to move as much as the dimensional restrictions allow.*
- Looking for patterns in the number of vertices, edges, faces, solids, etc in a point, line segment, square or triangle, cube or tetrahedron, and using the patterns to predict how many vertices, edges, faces, solids, 4-d regions and 5-d regions are in the hypercube, hyperhypercube, 6th dimensional cube, etc or hypertetrahedron, hyperhypertetrahedron, 6th dimensional tetrahedron, etc.
- Ways to represent the hypercube in 2 and 3 dimensions.
- Which of Euclid's five postulates are we discarding in non-Euclidean geometry? Be able to state this postulate. (It's on one of the webpages available through my unlinked website.)
- Describe a model of elliptic space, and what lines and circles look like on this model.
- Discuss triangles in elliptic space (what they look like, the sum of their angles, and their areas).
- Be able to describe at least one model of hyperbolic space.
- What do lines look like on the Poincaré disc model of hyperbolic space?
- Name an artist who was interested in hyperbolic space and did several works of art relating to it. (Duchamp, Escher)

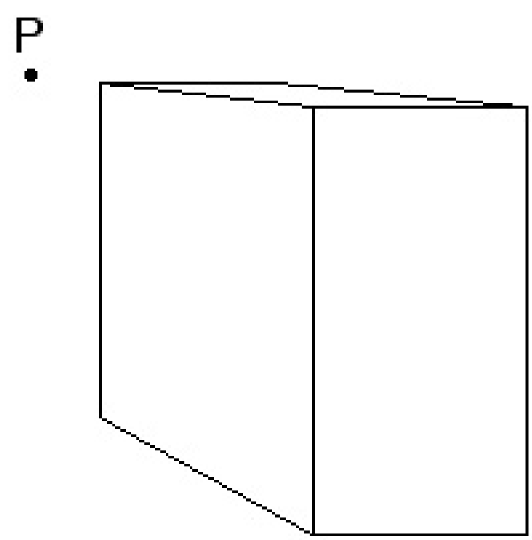
- Problems:

  The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

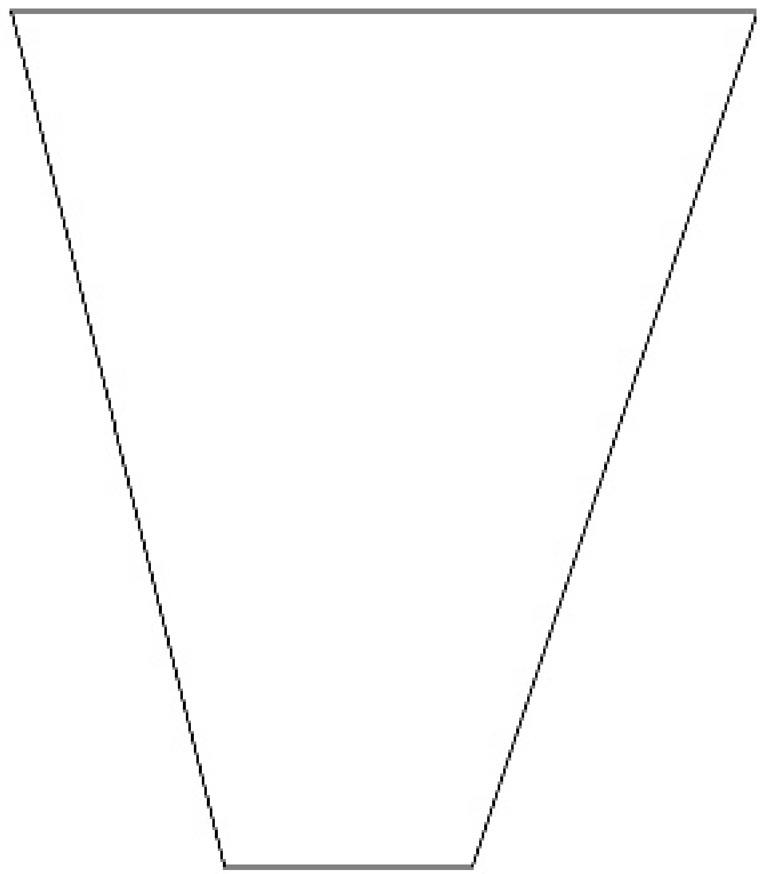
  1. Below is a perspective drawing of a window, retreating orthogonally to the picture plane. Draw a duplicate of this window, so that it's upper rear corner is located at the point P, to create the appearance of a partially open sliding glass door.



2. Below is a perspective drawing of a box, along with a point P. Draw a duplicate of the box, using the techniques we've developed. Place the duplicate so that it's front left corner (as we face it) is located at the point P, to create a picture of two boxes separated by some space.



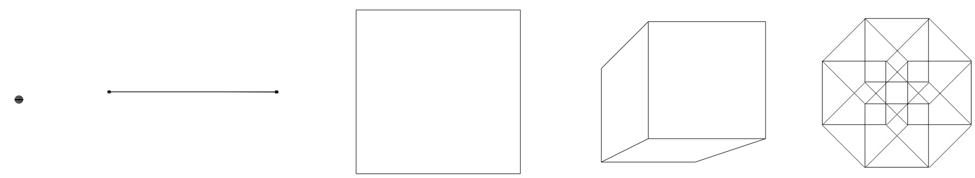
3. On the perspective drawing of a rectangle below, draw a horizontal line cutting the sides which no longer appear parallel into the division one-ninth/eight-ninths, without using a ruler. Probably the easiest way to do this, using the "stacking rectangle" technique we developed for the homework, is to divide the rectangle into thirds, and then one of the thirds on an end into thirds again.



4. What would A. Square observe (assuming he has the leisure to walk around and perhaps even touch), if a cube passed through Flatland edge first? (find a cube and experiment; we saw this in the movie on the hypercube but I don't expect you to remember it)

(15 pts) 5. Recall the creation of a line segment, square, cube, and hypercube:

0D point	1D line segment	2D square	3D cube	4D hypercube



Pondering the creation of the hypercube and the hyperhypercube resulted in the following table, up through dimension 5. Complete the final column for the sixth dimensional hyper-hyper-hypercube. You may use formulas we developed or the process of creating the hyperhyperhypercube, but however you do it, give some brief explanation.

Dimension Figure	0D point	1D line segment	2D square	3D cube	4D hypercube	5D hyper-hypercube	6D hyperhyper-hypercube
vertices v	1	2	4	8	16	32	
edges e	0	1	4	12	32	80	
faces f	0	0	1	6	24	80	
solids s	0	0	0	1	8	40	
4D regions t	0	0	0	0	1	10	
5D regions u	0	0	0	0	0	1	
6D regions w	0	0	0	0	0	0	

6. We can define something called the *Euler characteristic* of any figure in any dimension. For figures in five dimensions or less, the Euler characteristic is defined as follows:

$$\text{Euler characteristic} = \chi = v - e + f - s + t - u,$$

where v represents the number of vertices of the figure, e the number of edges, f the number of faces, s the number of solids, t the number of 4-dimensional regions, and u the number of 5-dimensional regions.

Find the Euler characteristic χ of each of the figures listed in the table in the previous problem. Notice anything?

7. Use ideas and/or formulas developed in class to decide what the most likely intersection of a plane with a 3-space is in 4-space.

8. What does a line on the sphere (one of our models of elliptic space) look like? Is there a unique line through *any* two points on the sphere?

9. Does the concept of parallel lines exist in elliptic space? If not, why not? If so, can there be two lines through the same point that are *both* parallel to a third line not through the point? (This is impossible in Euclidean space).

10. Is there a unique line through *any* two points in hyperbolic space?

11. Does the concept of parallel lines exist in hyperbolic space? If not, why not? If so, can there be two lines through the same point that are *both* parallel to a third line not through the point?

12. What can you say about the area of a circle in elliptic space? In hyperbolic space?

    (a) Find a triangle on the surface of the sphere whose angles add up to 270°. Is the area of this triangle equal to, greater than, or less than

    $$\frac{1}{2} \times \text{base} \times \text{height?}$$

    (b) Can you find a triangle on the sphere whose angles add up to less than 180°?