1. Suppose you have a line segment AB.

   (a) If you have a line segment AB, where on it must you put the point C so that the ratio $\frac{\overline{AB}}{\overline{AC}}$ is 2? When C is in this location, what is the value of the ratio $\frac{\overline{AC}}{\overline{BC}}$?

   (b) If you have a line segment AB, where must you put the point C so that the ratio $\frac{\overline{AB}}{\overline{AC}}$ is 1? When C is in this location, can you find the value of the ratio $\frac{\overline{AC}}{\overline{BC}}$? If not, why not?

   (c) Based on your responses to Exercises 1a and 1b, what are the smallest and largest possible values for the ratio $\frac{\text{length of the whole}}{\text{length of the greater}}$?

   Even without calculating the Extreme and Mean ratio, this gives us a range that we know the Extreme and Mean ratio falls within.

2. For each of the following, you'll be drawing a line that is cut in Extreme and Mean ratio (i.e. the Golden Ratio).

   (a) Suppose we want to draw a line cut in Extreme and Mean Ratio, and we want the short segment to be 3 units long. How long should the long segment be? Draw such a line as carefully as possible.

   (b) Suppose we want to draw a line of length 6 that is cut in Extreme and Mean ratio. Where should we place the cut? Draw such a line, as carefully as possible.

   Look at the two lines with cuts that you've drawn. Do they look the same or different? (You don't necessarily have to address this in what you turn in, I just want you to pay attention to the big picture!)

3. Suppose we don't happen to agree with the ancient Greeks about balance between extremes. So we define our own standard of beauty and all that, beginning with the ideal way to cut a line. Here's ours:

   *A line is said to be cut in the **Very Cool Ratio** when the greater segment is to the lesser segment as **twice** the whole is to the greater.*

(a) Set up the proportion described in the definition of the Very Cool Ratio.

(b) What *is* this Very Cool Ratio? (By this I mean, *find* the number it equals!)

(c) Suppose we want to draw a line cut in the Very Cool Ratio, and we want the short segment to be 3 units long. How long should the long segment be? Draw such a line as carefully as possible.

(d) Suppose we want to draw a line of length 6 that is cut in the Very Cool ratio. Where should we place the cut? Draw such a line as carefully as possible.

(e) How do these cuts compare to those you found in the previous problem?

4. Wikipedia says that Salvador Dali "explicitly used the golden ratio in his masterpiece, *The Sacrament of the Last Supper* [shown below]. The dimensions of the canvas are a golden rectangle. " This painting is 267 cm wide and 166.7 cm tall.

(a) Find an acceptance range for the Golden Ratio, assuming the measurements are each accurate to within 0.5%. (You may assume that accuracy in measurements within 0.5% translates to accuracy in the ratio within 1%.

(b) Based on this margin of error and the resulting acceptance range found in part (a), do the measurements support this claim?

(c) What do you conclude from this?

Note: We know that pentagons give rise to the Golden Ratio exactly. Notice that a huge dodecahedron (which is composed of pentagons) is suspended above and behind Jesus. Wikipedia also says that the edges of it appear in golden ratio to one another. Check it out if you like!

5. You want to paint a picture on a rectangular canvas, and you want the ratio of the length of the long side to the length of the short side to be φ, the Golden Ratio – that is, you want the canvas to be in the shape of a *Golden Rectangle*. If the short side of your canvas is 2' wide, how long should you make it be?

6. You have read, in the excerpts from *The DaVinci Code*, that "my friends, each of you is a walking tribute to the Divine Proportion." In this exercise, you will explore whether *you* are a such a tribute to the Golden Ratio.

   (a) Measure the following. In each case, give an accuracy range based on your sense of how accurately you (or a friend) measured that quantity.

      i. your height

      ii. the distance from your navel to the floor

      iii. the distance from your shoulder to your fingertips

      iv. the distance from your elbow to your fingertips

      v. the distance from your hip to the floor

vi. the distance from your knee to the floor

Make sure your lengths are not in mixed units like feet and inches. For instance, rather than listing my height as $5'3.5" \pm .5"$, I would write it as $63.5" \pm .5"$. Also, use the same units throughout your measurements. Label each your measurements very clearly.

(b) *Height to belly button height:*

i. Calculate the ratio of your measured height to the measured height of your belly button.

ii. Calculate the range that *actual* ratio might fall into, using the ranges you found above.

iii. Does the Golden Ratio fall into this range?

(c) *Arm length to fore-arm length:*

i. Calculate the ratio of your measured arm length to your measured fore-arm length.

ii. Calculate the range that this actual ratio might fall into.

iii. Does the Golden Ratio fall into this range?

(d) *Leg length to height of knee:*

i. Calculate the ratio of your measured leg length to the measured height of your knee.

ii. Calculate the range that this actual ratio might fall into.

iii. Does the Golden Ratio fall into this range?

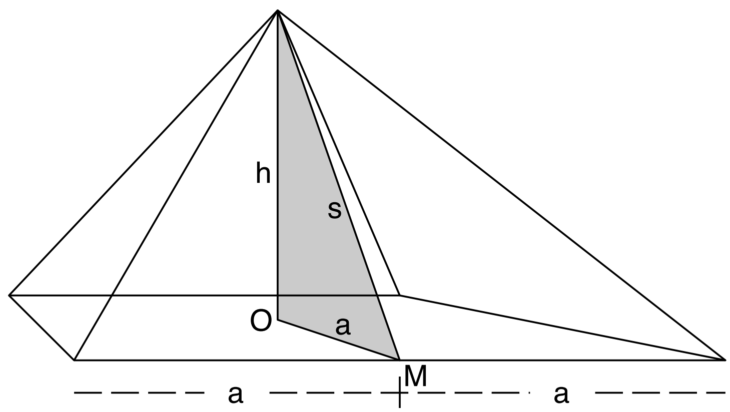
(e) Draw some conclusions as to whether you believe *you* are the tribute to the Divine Proportion that Dan Brown's Robert Langdon claims you are.

7. You have read, in Section 2.2.3, that it is frequently said that Herodotus described the construction of the Great Pyramid by saying that the Pyramid was built so that the area of each face would equal the area

of a square whose side is equal to the Pyramid's height. You have also read that this relationship would ensure that the Golden Ratio appear in the Great Pyramid, but I left out some details for you to work out in this exercise.

Let

$$\begin{aligned} h &= \text{height of pyramid} \\ a &= \frac{1}{2}(\text{length of base}) \text{ -- so } 2a = \text{length of base} \\ s &= \text{slant height} = \text{the height of a triangular face} \end{aligned}$$



(a) Find the area of a square whose sides all have length h (the height of the pyramid).

(b) Find the area of one of the triangular faces of the Pyramid.

Remember: Area of a triangle=$\frac{1}{2}$(base) × (height).

(c) Rewrite the statement attributed to Herodotus, using the expressions for area you found in parts 7a and 7b.

(d) By looking at the above diagram of the pyramid, find another equation that connects h, s, and a.

Hint: Look for a different triangle!

(e) Combine these two equations in a logical way to find a relationship between a and s. Solve for s/a. (You should get that $s/a = \varphi$!)

8. You have also read in Section 2.2.3 that if the Egyptians used rollers to measure the length of the base of the pyramid, and ropes to measure the height of the pyramid, then π would have been sure to appear in the Great Pyramid. I again left the details to this exercise.

   (a) Suppose you build a model of a pyramid as follows: take a wheel of diameter d and lay out a square base whose sides are each one revolution of the wheel long. Then make the pyramid height equal in length to two diameters of the wheel.

      i. How long is the base of your model? (Your answer will be in terms of d.)

         Remember: Circumference of a circle$= 2\pi \times$ *radius* $= \pi \times$ *diameter.*

      ii. How tall is your model? (Again, your answer will be in terms of d.)

      iii. Find the ratio of the height of your model to the length of the base of your model.

      iv. Find the ratio of the height of the Great Pyramid to the length of the side of the base of the Great Pyramid.

         Recall: The height of the Great Pyramid is 481.4 feet and the length of the side of the base of the Great Pyramid is 755.79 feet.

      v. Draw some conclusions about the shape of the Great Pyramid and the shape of your model.

   (b) In this part of the problem, you're going to show that the Egyptians wouldn't have had to use a gigantic measuring wheel for this process to have worked.

      i. Suppose you lay out a square base whose sides are each 10 revolutions of the wheel long, and you make the height be 20 diameters of the wheel. Find the ratio of the height of this new model to the base of this new model. How does it compare to the ratios you found in the previous parts?

      ii. Suppose you lay out a square base whose sides are each n revolutions of the wheel long, and you make the height be $2n$

diameters of the wheel tall. Again, find the ratio of the height of this new model to the base of this model, and compare.

(c) Using the dimensions for the Great Pyramid given above, find the diameter of the measuring wheel required so that 100 revolutions of the wheel would produce one side of the base of the Great Pyramid and 200 diameters would give the height. Is this a reasonable sized for the measuring wheel? That is, is it likely the Egyptians would use a measuring wheel this size, if they constructed the pyramid this way?