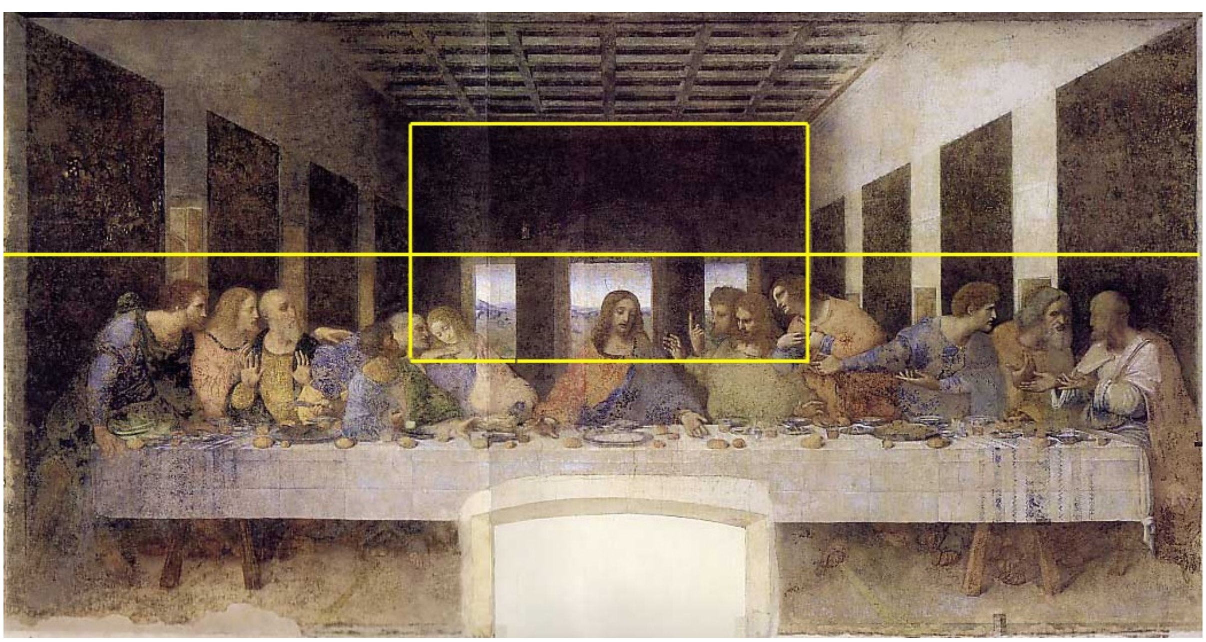
1. In this problem, you will be studying Leonardo da Vinci's *Last Supper*, and (ultimately) deciding whether you believe he was intentionally trying to incorporate the Golden Ratio. Measure as carefully as you can.

   (a) Find the acceptance range for φ based on a 2% margin of error in measurement (which will allow for a 4% margin of error in your ratios).

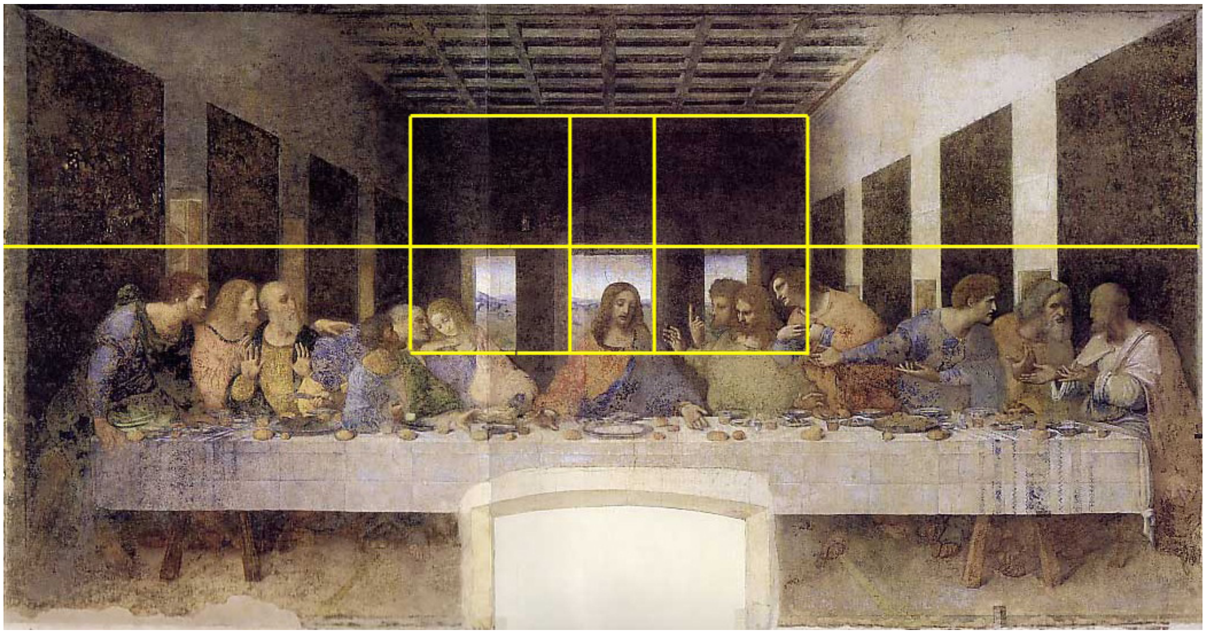
   (b) Below, I have super-imposed a horizontal line that follows the tops of the insides of the windows on the rear wall all the way to the edges of the mural. Does that line cut the height of the painting into Extreme and Mean Ratio?



(c) In the figure on the previous page, I have also outlined the rectangle formed by the rear wall, down to the bottoms of the windows. Is it a Golden Rectangle (within our margin of error)?

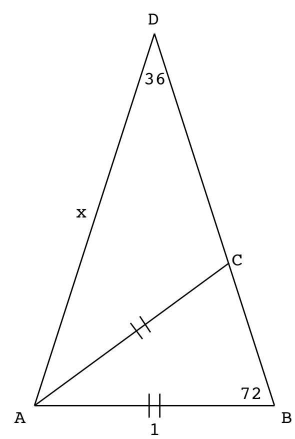
(d) Also in that figure, does the long horizontal line cut the height of the rear-wall rectangle in Extreme and Mean Ratio?

(e) On the next page, I have super-imposed two vertical lines that upwardly extend the vertical lines that define the central window, forming rectangles on the left and right that are subdivided into a top part and a smaller rectangle below. Carefully measure both of the larger rectangles (the ones that go from the top of the wall down to the bottom of the windows, and from the edges of the walls to the window behind Jesus), and determine whether they are Golden, within our acceptance range.



(f) In the same figure, carefully measure both of the smaller rectangles (to the left and right of the central window), and determine whether they are Golden (within our margin of error).

2. *Golden Triangles:* In this exercise, you will be showing that all of the triangles shown in the triangle below have the ratio *long side*:*short side* in the Golden Ratio. Since any triangles with the same angles will be similar, and hence have sides in the same proportions, this will ultimately show that any 72°-72°-36° triangle and any 36°-36°-108° triangle will be *Golden*, although you will of course first have to discover those angles. (Also, any isosceles triangle whose sides have this ratio will be similar to one of these two types.)



(a) Show that triangle ABD is similar to triangle BCA.
Hints:
- Since you're given that $\overline{AB} = \overline{AC}$, you know that triangle BCA is an isosceles triangle.
- Remember that the base angles of an isosceles triangle are equal, *and* if the base angles of a triangle are equal, it's an isosceles triangle.
- Although triangle ABD *looks* like an isosceles triangle, you are not given that it is.
- Also remember that the sum of the angles in a triangle are 180°.

(b) Show that triangle ADC is also an isosceles triangle.

(c) Use the result of part (b) to write the length $\overline{BC}$ in terms of x.

(d) Use the similarity you showed in (a), and your result to part (c), to show that $x = \varphi = \dfrac{1+\sqrt{5}}{2}$.

(e) Show that in the isosceles triangle ADC, the ratio of the longer side to the shorter side is again φ.

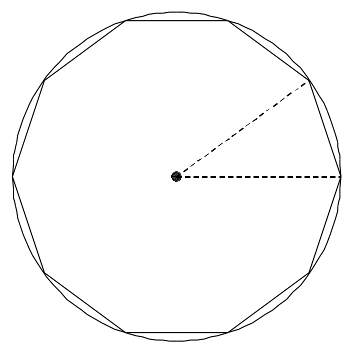
3. I found a couple of websites that claim that Raphael's *Mond Crucifixion* exhibits a Golden Triangle. Below is a replica of the diagram I found on the web.

   (a) By carefully measuring this diagram, find the ratio of the length of the long side of this triangle to the base.

   (b) Assuming a 2% margin of error in your measurements, does this triangle fall within the margin of error being a Golden Triangle?

(c) Do you think the placement of this triangle is defined in a natural or obvious way by the features of the painting, or do you think it was conveniently chosen to create a nearly golden triangle? In case the lines obscure some of the detail: the top vertex is at the point where the horizontal line defined by the bottom of the crossbar intersects the midline defined by Jesus' body; this point is also the rightmost edge of the crown of thorns. The left side of the triangle extends to follow the arm of St. Jerome (kneeling), while the right side extends to follow the cloak of John the Evangelist (standing). The horizontal leg just grazes the robe of St. Jerome on the left and the cloak of Mary Magdalene on the right.

4. *Another relationship between φ and π:* A regular decagon (that is, a figure with 10 equal sides and 10 equal angles) can be inscribed in a circle of radius r, as shown below. Using $r = 1$, to make the calculations simpler,



(a) find the perimeter of the decagon in terms of φ, using the results of Problem 2.
Hint: You'll need to find the length of the sides of the decagon. Since a full circle is 360°, can you figure out what the angle I've shown at the center is?

(b) use that the perimeter of the decagon and the circumference of the circle are roughly equal to find an approximate expression that relates φ and π.

5. We know that the first 10 Fibonacci numbers are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. Remember that we use the notation F_n to represent the nth Fibonacci number – that is, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc. Also remember that we find the nth Fibonacci number by adding together the two that come before it. Find the numerical value of the following:

   (a) F_{11}

   (b) $F_{11} + 2$

   (c) F_{11+2}

6. Below is a list of works of art often said to incorporate the Golden Ratio. Please pick one (or more), and photocopy it from a book. (Avoid print-outs from the web, as they can be distorted in shape, and so your results will not mean much.)

   - Dürer's *Adoration of the Magi*
   - Da Vinci's *Mona Lisa*, *St. Jerome*, *A Head of an Old Man*, or *Annunciation*
   - Michelangelo's *David*
   - Seurat's *The Bathers*
   - Mondrian's *Place de la Concorde*
   - Gris' *The Watch* (aka *The Sherry Bottle*)

   Once you've chosen your painting,

   (a) Really look at it, and try to think of as many ways as possible that the Golden Ratio may have been used.

   An artwork may incorporate the Golden Ratio in many ways. One obvious way would be if the painting itself were a Golden Rectangle, but there are lots of other possibilities:

   - a line in the painting may be cut in the Extreme and Mean Ratio
   - a rectangle in the painting may be a Golden Rectangle. Such a rectangle may be explicit, or it may be formed by using four features in the painting as corners.
   - an isosceles triangle in the painting may be a Golden Triangle (that is, the ratio of long side to short side may be the Golden Ratio). Again, such a triangle may be explicit, or three features may form the corners.
   - a rectangle fitting snugly around a figure in the painting may be a Golden Rectangle, and similarly for a Golden Triangle

- a body might have been drawn so that various parts are in the Golden Ratio
- the ratio of two distances between items may be the Golden Ratio, etc

Feel free, by the way, to read up on the claims relating to your choices on the web or in books. Just make sure you do the measuring yourself. Draw any lines as thinly as you can, as you're dealing with a much smaller version than the original, so a thin line on a shrunk version would correspond to a very thick line on the original.

(b) Measure all the distances you thought of in the previous part. Clearly label your measurements, and (as indicated in the previous part) draw the lines on the photocopy you're using.

(c) Decide upon an accuracy range for your measurements.

(d) Calculate all the ratios you thought of in the first part, including acceptance ranges for each.

(e) Does the Golden Ratio fall into any of these?

(f) Do you think the artist had the Golden Ratio in mind when creating this artwork? (This may not follow immediately from your previous results if, for instance, you think the Golden Ratio was close enough that it might have been intentional, even if it didn't fall into your ranges. After all, your ranges only reflect your measurement range, not any adjustments or errors the artist might have made.)