1. Given that $F_{36} = 14,930,352$ and $F_{37} = 24,157,817$, find:

   (a) F_{38}

   (b) F_{35}

2. Let a represent the 1000th Fibonacci number and b represent the 1001st Fibonacci number. Express the 1003rd Fibonacci number in terms of a and b. (In other words, you're doing this without ever knowing what the 1000th and 1001st Fibonacci numbers are.) Simplify your answer.

3. Binet's Formula defines Fibonacci numbers directly:

$$F_N = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^N - \left(\frac{1-\sqrt{5}}{2}\right)^N}{\sqrt{5}}.$$

Use Binet's Formula to find the 15th Fibonacci number.

Hint: Don't try to do this by hand - -use a calculator; specifically, use the kind of calculator that allows you enter a long expression with parenthesese. Be very careful with your parentheses! If you do it all on the calculator (that is, without writing down rounded intermediate results and then re-entering those values later), you *should* end up with a whole number, with no rounding necessary, which always seems miraculous!

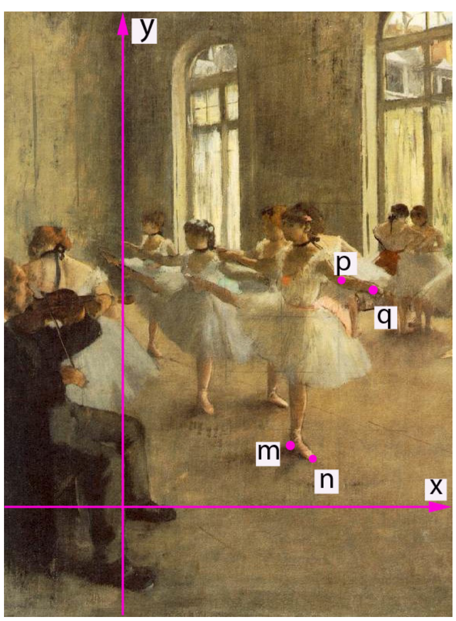
(Many of the rest of the problems are either taken directly from *Lessons in Mathematics and Art*, Lessons 1 and 2 or modified from it).

4. Below is a detail from Edgar Degas' painting, *The Rehearsal*, with 2-dimensional coordinate axes superimposed on the picture plane. Using the points $m(216, 88)$, $n(249, 68)$, $p(283, 302)$, and $q(317, 293)$ in the figure (the coordinates are in pixels), find the following distances:

   (a) $d(m, n)$

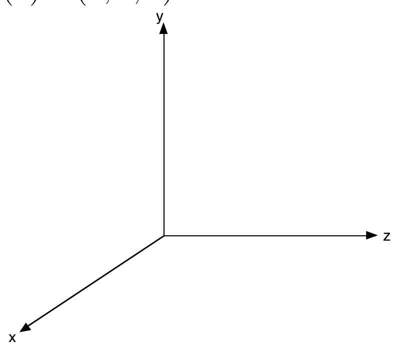
   (b) $d(p, q)$

   (c) Several systems of proportions dictate that a person's foot should be about the same length as their forearm. Thinking of the painting as a window onto a "real" dance studio, it looks as if the actual three-dimensional dancer's left foot and left forearm would be roughly parallel and directly above one another. We'll see later that because of that, if they were indeed the same length in real life, then their *images* in the painting would also be the same length. *Are* these images the same length?

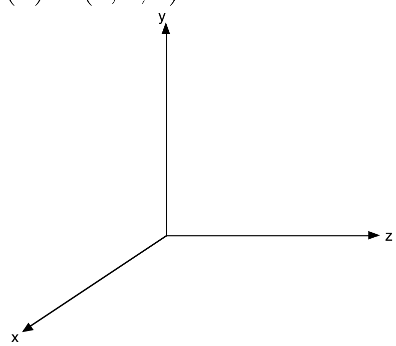


5. Please plot the following points on a set of 3-D coordinate axes (using the coordinate system used for this course rather than the standard coordinate system.) Mark units on your axes, and show enough dashed lines so I can see how you found where to put your points.

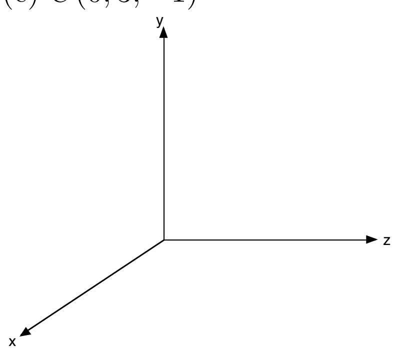
   (a) $A(1, 3, 4)$



   (b) $B(2, 4, 0)$



   (c) $C(0, 3, -1)$



6. Assume a viewer who is located with eye on the negative z-axis is looking at two points $A(3, 3, 2)$ and $B(4, 2, 7)$.

   (a) Which is higher?

   (b) Which is closer to the viewer?

   Note: Usually, to determine which is closer to the viewer, we would need to know the viewer's exact location and then use the distance formula to calculate how far the viewer is from each point. However, in this case one point is *much* closer to the picture plane, and hence closer to the viewer on the other side of the picture plane, than the other. Which?

   (c) Which is further left, to the viewer?

7. In this problem, you're going to be considering a box whose faces are parallel to the coordinate planes. Suppose the corners of this box have the following coordinates:

Bottom	Top
$A(1,3,4)$	$E(1,7,4)$
$B(8,3,4)$	$F(8,7,4)$
$C(8,3,10)$	$G(8,7,10)$
$D(1,3,10)$	$H(1,7,10)$

Note 1: You do not have to plot each of the 8 corners and draw the box in order to do this question, although you certainly can if you think that will help you think about the problem clearly.

Note 2: When using words like "wide" and "deep" below, I am thinking about how the box would appear to a person with their eye at our assumed viewing position on the negative z-axis.

(a) How wide is the box (that is, in the x direction)?

(b) How tall is the box in the y direction?

(c) How deep is the box (that is, how far back in the z direction does it go)?

8. Now I want you to look for patterns in Problem 7, and use them. We again have a box whose faces are parallel to the coordinate planes. Suppose the coordinates of two opposing corners of a box (the front-left-bottom corner and the rear-right-top corner) have coordinates $(3, -1, 5)$ and $(10, 7, 10)$.

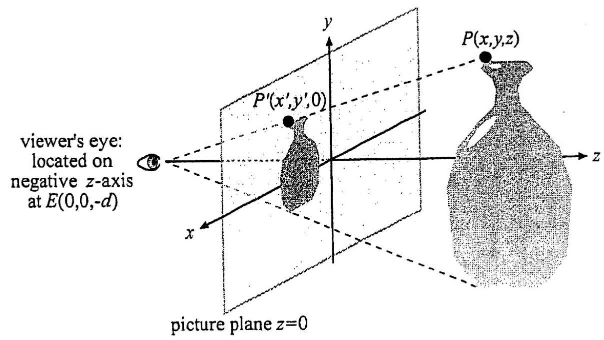
   (a) How wide is the box, to the viewer?

   (b) How tall is the box?

   (c) How deep is the box, to the viewer? (That is, how far back does it go?)

   (d) Use the insights gained from parts (a) through (c) to determine the coordinates of the remaining 6 corners.

9. In the figure below (from Lesson 1 of *Lessons in Mathematics and Art*), suppose that $d = 3$ and suppose that the point $P(x, y, z)$ were moved so that $x = 0$, $y = 4$, and $z = 5$.



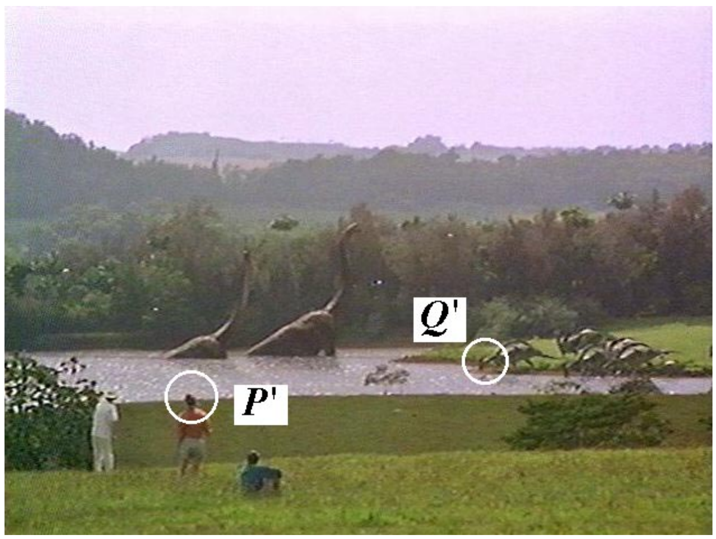
(a) Which coordinate plane would the point $P(x, y, z)$ lie in?

Remember: The *coordinate planes* are the planes determined by the three possible pairs of axes. The xz-plane is the horizontal plane that you might think of as the "floor"; the xy-plane is the vertical plane that we use as the picture plane -you might think of it as a window; the yz-plane is the vertical plane that we might think of a side wall (in this picture, it appears to *us* to be behind the vase, although to the viewer it would be a wall directly in front and extending away).

(b) Without using the Perspective Theorem, what would x', the image of x in the picture plane, be? (You can always check your answer using the formula, of course!)

(c) Again without using the Perspective Theorem, what would y', the image of y in the picture plane, be?

10. Think of the Jurassic Park image in Figure 3 of Lesson 2, included below, as being an image of a *real* scene painted onto a picture plane. Let $P(x, y, z)$ be the top of the woman's head in real life (*not* in the image), and let $Q(x, y, z)$ be the top of the head of the dinosaur drinking water in real life. The circled and labeled points P' and Q' are the respective *images* of these points.



Which is bigger:

(a) the x-coordinate of P, or the x-coordinate of Q?

(b) the y-coordinate of P, or the y-coordinate of Q?

(c) the z-coordinate of P, or the z-coordinate of Q?

(d) the x-coordinate of P', or the x-coordinate of Q'?

(e) the y-coordinate of P', or the y-coordinate of Q'?

(f) the z-coordinate of P', or the z-coordinate of Q'? (This is a trick question)

11. This exercise deals with a point P that moves farther and farther away from the picture plane and the viewer, without moving left or right, up or down. That is, the x- and y- coordinates of P do not change (they are equal to 2 and 3, respectively), but its z-coordinate gets bigger and bigger.

    Throughout this problem, use that the viewing distance d is 5 units and use the Perspective Theorem.

    (a) Suppose $P = (2, 3, 5)$. What are the values of x' and y'?

    (b) Now suppose $P = (2, 3, 95)$. What are x' and y'?

    (c) What if $P = (2, 3, 995)$?

(d) Draw one TOP VIEW and one SIDE VIEW like those we did in class (they're also in Figure 2 of Lesson 2 in *Lessons in Mathematics and Art*), and include all the points P and P' from parts (a)-(c), along with light rays to the viewer's eye (the drawings need not be to scale). Can you see what's happening?

(e) Consider a point $P(x, y, z)$. If x and y do not change, but z gets bigger and bigger, what happens to the picture plane image P' of P?

(f) Our everyday experience tells us that objects appear smaller as they get farther away. Explain how this is consistent with your answers to parts (a)-(e).