Part I

In this portion of the assignment, you are going to draw the same cube in different positions, using the Perspective Theorem. You will then use these pictures to make observations that should reinforce the conclusions about the perspective images of various types of lines that are being discussed in class. You *will* need graph paper! (I would suggest having each unit be several squares long, so your pictures are big enough to really appreciate.)

In both cases, our cube will have side of length 4, and the viewing distance d (how far the viewer's eye is from the picture plane) will be 8.

We will call our cube $ABCDEFGH$ with the base being square $ABCD$ and the top being square $EFGH$. (Note that in the base square, A is connected to B and D, B is connected to A and C, etc; and that E is directly above A, F is directly above B, etc).

1. What are the coordinates for the viewer's eye?

2. We'll begin with a cube whose top and bottom are horizontal and whose front and back are parallel to the picture plane. The bottom will be above the viewer's eye.

   Use the following coordinates for the corners of the cube:

Base=ABCD		Top=EFGH	
A	$(8,3,4)$	E	$(8,7,4)$
B	$(12,3,4)$	F	$(12,7,4)$
C	$(12,3,8)$	G	$(12,7,8)$
D	$(8,3,8)$	H	$(8,7,8)$

   (a) Using the Perspective Theorem, find the coordinates for each of the 8 corners (shown again below) of the image in the picture plane (that is, find (x', y')). You may do these calculations by hand or, if you're comfortable with it, you may use a spreadsheet like Excel. If you do it by hand, include your work on a separate sheet; if you use a spreadsheet, please include it with your work.

(b) *Carefully* plot the points you found in part 2a in the xy plane on graph paper. (Remember you are *not* using 3D space axes for this!) Then (paying attention to the right order), neatly connect them with straight lines (use a straight edge, and use dashed lines to indicate the hidden edges) to obtain the perspective image.

(c) Get a good idea of what the viewing distance is in the scale you used (that is, how long is 8 units?), and then put one eye at that distance from the page, directly opposite the origin. Look at your perspective image with that one eye. Do you see a cube, with the illusion of depth?

(d) Your cube has one set of four parallel lines which are not parallel to the picture plane. Do those lines look parallel in the perspective image you've created? Using however many straight edges (pieces of paper, for instance) you need, see where they intersect (this may not be on your piece of graph paper). What can you say about where these four lines intersect?

3. We'll continue with the same cube, but we'll turn it so that while the top and bottom are still horizontal, now one edge is facing us, rather than the front and back being parallel to the picture plane. We'll also move it so that the top is below the viewer's eye.

   Use the following coordinates for the corners of the cube:

Base=ABCD		Top=EFGH	
A	$(0,-6,4)$	E	$(0,-2,4)$
B	$(2.8,-6,6.8)$	F	$(2.8,-2,6.8)$
C	$(0,-6,9.7)$	G	$(0,-2,9.7)$
D	$(-2.8,-6,6.8)$	H	$(-2.8,-2,6.8)$

(a) Find the coordinates for each of the corners of the image in the picture plane (include your work or spreadsheet). Carefully plot them in the xy plane on graph paper, then neatly connect them with straight lines to obtain the perspective image.

(b) As with the previous exercise, put one eye at the viewing distance opposite the origin. Look at your perspective image with that one eye. Does it leap off the page at you?

(c) Again, can you get a sense of where the parallel lines intersect?

Part II:

For the next 3 problems, you will be investigating the use of perspective in 3 paintings by Renaissance masters. You will need print-outs of each of these paintings; because you may need several of each, I have put links to the paintings with the problem set rather than include the pictures in this file. There's no need for the print-outs to be in color, although you may enjoy the process more. Hand in the print-outs (along with any additional pieces of paper that you needed), as this is where most of your work for these problems will be. You may find you need a couple print-outs of each painting.

4. Consider Leonardo's *The Last Supper* (1495-1498), 460 cm $\times$ 880 cm.

   (a) **Finding the primary vanishing point**

      i. Locate and highlight at least two lines parallel to the picture plane and parallel to each other.

      ii. Locate and highlight in a different color (or on a different print-out) at least three lines orthogonal to the picture plane. (Remember, in real-life, these would be perpendicular to lines you found in part (a) and be receding).

iii. Extend the lines you found in part (a) to find the primary vanishing point. In my experience, this may take several tries, as it can be difficult to line your straight-edge up exactly with an orthogonal, and some may be off initially.

iv. Is the primary vanishing point in the picture or off the picture? Is it used to draw the eye anywhere important, or is it just used to give an illusion of depth?

(b) **Finding the ideal viewing position of your print-out**

i. Draw the horizon line through the primary vanishing point.

ii. Locate a secondary vanishing point by finding a square lying parallel to the floor, drawing its diagonal, and extending that diagonal until it intersects the horizon line.

iii. Determine the intended viewing distance for your print-out by measuring the distance between the primary and secondary vanishing point.

iv. Determine the ideal viewing position for your copy, and describe it.

v. Try viewing it from the correct viewing position. Does it improve the illusion of depth in the picture? (Of course, with a print-out it's not the same experience as looking at it in person.)

(c) **Estimating the ideal viewing position for the painting itself**

i. Measure the height and width of your print-out.

ii. Use the measurements, along with the dimensions of the original painting, the viewing distance you found in (b), and your knowledge of proportion to get a pretty good estimate of the ideal viewing position of the actual painting.

5. Next, consider Rafael's *School of Athens* (1509-1511), 500 cm × 770 cm.

(a) Find the primary vanishing point, by first locating at least two lines that are parallel to the picture plane and parallel to each other and then using those lines and your experience to deduce at least three lines that must be orthogonal to the picture plane. Is the primary vanishing point in the picture or off the picture? Is it used to draw the eye anywhere important, or is it just used to give an illusion of depth?

(b) Finding the ideal viewing position of your print-out. Again, try viewing your print-out from that position – does it improve the illusion of depth?

(c) Use the dimensions of your print-out, along with the dimensions of the original painting, the viewing distance you found in (b), and your knowledge of proportion to get a pretty good estimate of the ideal viewing position of the actual painting.

6. Finally, consider Masaccio's *Trinity* (1427-1428), 667 cm *times* 317 cm, the painting that motivated our looking into finding the correct viewing position.

   (a) Find the primary vanishing point. Is it used to draw the eye anywhere important?

   (b) Find the ideal viewing position of your print-out; does viewing your print-out from that position improve the illusion of depth?

   Note: Finding a square to work with takes a bit more work in this painting. Try looking at the top of the columns. You will have to finish off the squares for yourself – be sure to use the vanishing point to draw in the missing orthogonal.

   (c) Use your results and the dimensions of the painting to estimate the ideal viewing position of the actual painting.

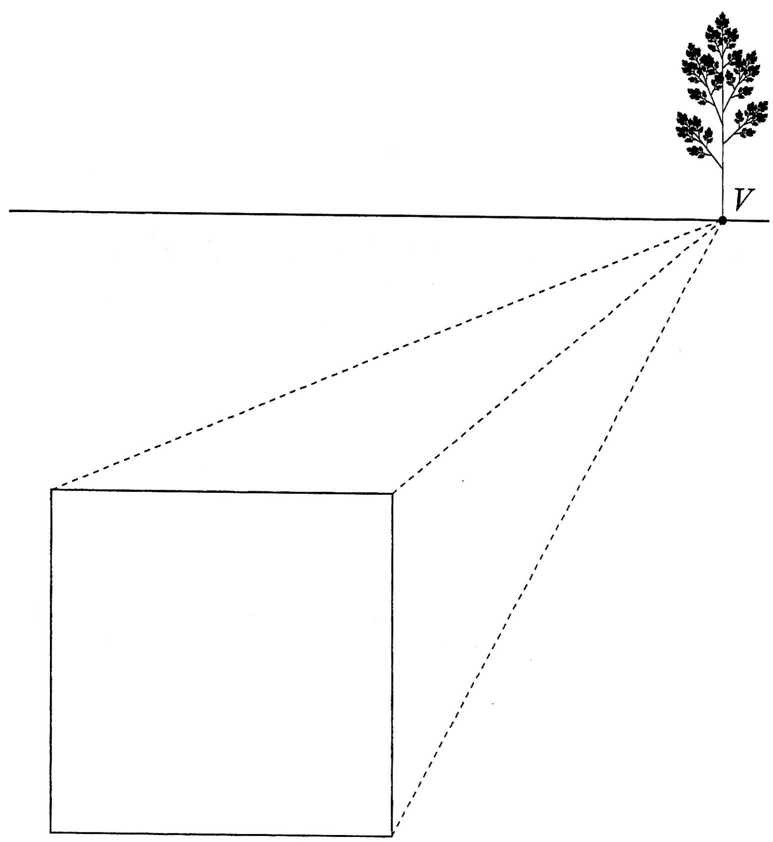
Part III:

These final two exercises are from *Lessons in Mathematics and Art*; they extend the ideas we used to come up with the ideal viewing position to different situations.

7. *Drawing your own cube:*

   In the figure below, a start has been made on the drawing of a cube in one-point perspective. The front face is a square, V is the vanishing point, and the dashed lines are guidelines for drawing receding edges of the cube. Suppose you want to choose the viewing distance *first*, and you choose it to be 6 inches. Finish drawing the cube.

   Hint: For help in thinking about it, look at Figure 7 from Lesson 3. The idea is to, in a sense, work backwards.



8. If the box below represents a cube, then we can use our usual techniques to find the correct viewing distance, and it would end up being the distance between the two trees, as illustrated.

   But suppose the box below is *not* a cube – suppose its front is a square, but its top face is in reality twice as long as it is wide from left to right. In this case, the viewing distance is *not* equal to the distance between the two trees. What *is* the viewing distance in that cae? (You may give your answer in terms of the distance between the trees, if that's easier.)

   Hint: Go through the same process we went through in class to find the viewing distance with a cube, but make appropriate adjustments.