- Exam 1 will will take place in class on Wednesday 2/23/11, and will cover through Problem Set 4 (that is, through gnomons.) To ensure everyone has the same amount of time (except for people who have already given me an accomodations letter), I will begin the exam promptly at 9:30 and will collect it at 10:27.

- Solutions to Problem Sets 1-3 are (or soon will be) on 2-hour reserve at the circulation desk in the library. I will put the solutions to Problem Set 4 and to this study guide on reserve Monday.

- I will include some of the formulas and proportions on the test, so that you do not need to memorize them. Specifically, I will give to you: the quadratic formula, the golden ratio, the sacred cut ratios, and Vitruvius' system of proportions

- Please don't take the following advice as the final word on how to study: everybody learns differently, after all. I would hate for someone to follow my advice to the letter (perhaps against their better judgment) and have it not work for them. This is what I've learned seems to work best for *most* people.

- Advice:

  - I know you have other classes, but spread studying for this exam out over several shorter study sessions, rather than planning on doing one or two extended study sessions. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc

  - In an ideal world, the best way to study for a math test is to re-read all the readings (including your notes – this course is definitely heavily notes-based!), summarize the topics we've covered, and re-do as many homework problems as possible.

    If you are not living in an ideal world (and who is?), I would still skim the readings, and in the notes from class try to emphasize connections with math and art that may not have been covered much in the readings. Unlike in many other subjects, however, your main focus, should be to *do* (not just read through) as great a variety of problems as possible. In addition to doing the problems I've included on this study guide, you'll also want to redo as many problems as you can from the first three problem sets. (Notice that I said "redo" – simply reading through solutions doesn't do it. )

  - When you're doing problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.

- When doing a problem that you've done before, don't waste your time trying to remember how you did it before–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense.
- Should you study alone or with other people? That varies from person to person, but in general I'd say most of your studying should be on your own, particularly as it gets closer to the day of the exam. For most people, I think group study is best at the beginning of the study process. Since the exam is individual, at some point in your studying, you have to be doing problems individually.
- How long should you study for this? Alot. "Alot" will vary from person to person also, but I'd suggest an absolute minimum of 6 hours. If you've struggled with the problem sets, then allow considerably more time. If you breezed through the problem sets, then you *may* be able to get away with less – but why risk it?!
- Get plenty of sleep the night before, and be sure to eat before taking the exam!

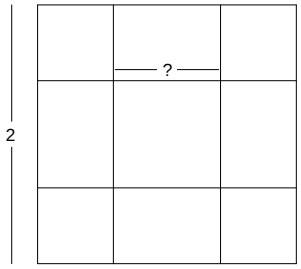
The following problems are intended as a supplement to your review; they are not intended to replace redoing homework problems or reviewing the reading and class notes.

**A word of caution:** You are responsible for all material covered in your reading, whether or not we covered it in class.

1. Use the Vitruvian system of proportion to decide how big you should make the distance from the middle of the breast to the crown if you've made the distance from the line between the eyes to the hairline 2" long.

2. Use geometry to find the length of the side of the Sacred Cut square, if the side of the outer square has length 2.

   *(This problem can be done using proportion; the reason I ask you to do it using geometry instead is because I want you to, in essence, re-develop the Sacred Cut proportions to make sure you understand the geometry behind them.)*




3. You are designing a web page with two pictures that you would like to place side by side. For aesthetic reasons, you'd like them to be the same height, and yet the way the pictures are currently stored, one is taller than the other.

   Here are the dimensions as they are stored on your computer:

| Picture | Width (pixels) | Height (pixels) |
|---|---|---|
| Canaletto's *Piazza San Marco Looking South-East* | 958 | 800 |
| Rafael's *School of Athens* | 800 | 554 |

   You need to make the taller picture be the same height as the shorter one, without changing its shape.

   (a) Which picture do you need to adjust?

   (b) How tall will it be?

   (c) Use proportion to decide how wide it should be.

4. Even after adjusting one picture, the pictures from the previous problem are way too wide. If you want the total width of the two pictures to be 800 pixels,

   (a) How high do they need to be, so that you haven't distorted either picture? (Use the results of part (a), so you only have to find one height.)

(b) How wide does each of the two pictures need to be, so that you haven't distorted either's shape?

5. Suppose Goldie measures her height and the height of her belly button. She only has a ruler, so the measurements aren't especially accurate: she finds her height to be $5'6" \pm 2"$ and the height of her belly button to be $3'3" \pm 1.5"$.

   Find the interval in which the *actual* ratio of her height to her belly button must lie. Does the Golden Ratio lie in this interval?

6. Suppose you know you've measured the height of an archway accurate to within 3% and the width of the archway accurate to within 1.5%. If you found the height to be 83" and the width to be 56",

   (a) find the interval in which the *actual* ratio of height to width must lie.

   (b) Is $\sqrt{2}$ in this interval?

7. For each of the following, you'll be drawing a line that is cut in Extreme and Mean Ratio (i.e. the Golden Ratio).

   (a) Suppose we want to draw a line cut in mean and extreme ratio, and we want the longer segment to have length 3. How long should the shorter segment be? Draw such a line as carefully as possible.

   (b) Suppose we want to draw a line cut in mean and extreme ratio, and we want the short segment to be 5 units long. How long would the whole line be? Draw such a line as carefully as possible.

8. Suppose a splinter group in the class neither agrees with Euclid nor with the Very Cool ratio advocates. They feel strongly that the most beautiful way of cutting a line is as follows:

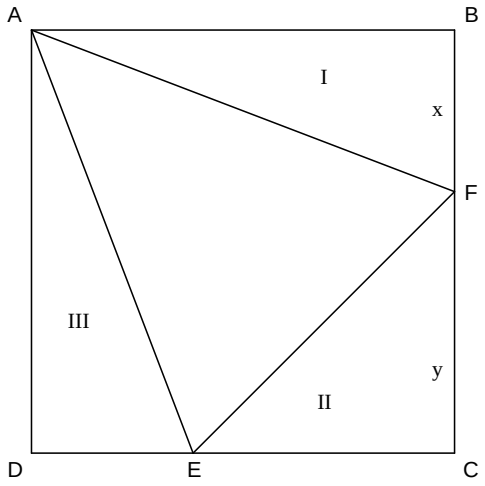
   *A line is said to be cut in an* **absolutely fabulous** *ratio when the greater segment is to the lesser segment as the whole segment is to twice the greater.*

(a) What *is* this absolutely fabulous ratio? (That is, find what number it equals)

(b) Discuss the issue of the short segment versus the long segment.

Since it turned out our ratio was 1-to-1, the only way that the whole can be to twice the greater as the greater is to the lesser is if the greater and the lesser are equal, and just divide the line in half. In other words, there isn't a "greater" and "lesser" segment, per se.

9. In the following figure, $ABCD$ is a square, and the three triangles $I$, $II$, and $III$ have equal areas. By following the steps below, you are going to show that $\frac{y}{x} = \varphi$.




(a) Find the area of triangle I, in terms of $x$ and $y$.

(b) Using that the areas of triangle I and triangle III are equal, find the length of $\overline{DE}$.

(c) Find the length of $\overline{EC}$.

(d) Find the area of triangle II.

(e) Using that the areas of triangle I and triangle II are equal, show that

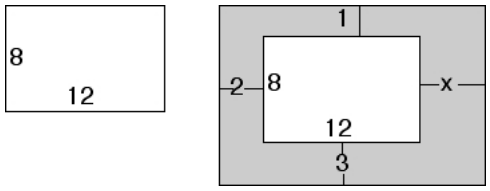
$$\left(\frac{y}{x}\right)^2 - \frac{y}{x} - 1 = 0.$$

(f) Use the quadratic formula to find $\frac{y}{x}$.

10. You showed in your homework that a pyramid created by measuring the base using some number of revolutions of the drum, and the height by using twice that number of diameters of the drum is very close to being similar to the Great Pyramid at Gizeh.

    While the Great Pyramid is the most famous of the pyramids, there are others at Gizeh (as well as throughout Egypt). The dimensions for the Second Pyramid at Gizeh are 470.75 feet high, with the sides of the base each being approximately 702' long. Is this pyramid also close to being similar to a pyramid that is constructed using the above technique?

11. Find the value of $x$ so that the shaded "rectangular ring" is a gnomon to the white rectangle.




12. Rectangle $A$ is 2 by 3. Rectangle $B$ is a gnomon to rectangle $A$. What are the dimensions of rectangle $B$?

13. A rectangle has a square gnomon. The new rectangle obtained by attaching the square gnomon to the original rectangle has longer leg 20. What are the dimensions of the original rectangle?