- Exam 2 will take place on Wednesday 3/30/11, and will cover through Problem Set 8 (which I'm just including at the beginning of this study guide, rather than having it be separate) – that is, it will cover through subdividing and duplicating rectangles in perspective.

- You may begin the exam at 8:30 am, or at 9:30 am (or anytime in between) – but I will collect it at 10:25 from all except those who have previously given me an accommodations letter and those who have an 8:30 class. *If you have an 8:30 class, please e-mail me – I'll figure out what to do on a case-by-case basis.*

- The solutions to Problem Sets 5-7 will soon be on 2-hour reserve at the circulation desk in the library (along with Problem Sets 1-4). I will put solutions to this study guide on reserve Monday.

- I will again be giving you a formula sheet along with your exam. This sheet will include the quadratic formula (just in case), the Golden Ratio, the first several Fibonacci numbers, and Binet's formula. **It will not include** the formula for finding a Fibonacci number from the previous two, nor will it include the formulas for calculating the coordinates of the perspective image of a point (that is, the perspective theorem).

- ADVICE: (Mostly the same as on the first study guide; the last item is new)

  - Let me emphasize this again –I know you have other classes, but spread studying for this exam out over several days. Information sinks in better; if you get frustrated, you can take breaks; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc

  - In an ideal world, the best way to study for a math test is to re-read all the readings (including your notes – this course is definitely heavily notes-based!), summarize the topics we've covered, and re-do as many homework problems as possible.

    If you are not living in an ideal world (and who is), I would still skim the readings, and in the notes from class try to emphasize connections with math and art that may not have been covered much in the readings. Your main focus, however, should be to *do* (not just read through) as great a variety of problems as possible. In addition to doing the few problems I've included on this study guide, you'll also want to redo as many problems as you can from the first three problem sets. (Notice again that I said "redo" – simply reading through solutions doesn't do it.)

  - When you're doing problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.

- When doing a problem that you've done before, don't waste your time trying to remember how you did before–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense.
- Should you study alone or with other people? That varies from person to person, but in general I'd say most of your studying should be on your own, particularly as it gets closer to the day of the exam. I think group study is best for most people at the beginning of the study process. Since the exam is individual, at some point in your studying, you have to be doing problems individually.
- How long should you study for this? Alot. "Alot" will vary from person to person also, but I'd suggest an absolute minimum of 6 hours. If you've struggled with the problem sets, then leave more. If you breezed through the problem sets, then you *may* be able to get away with less – but why risk it?!
- If you can not do the problems from start to finish without getting help from friend, tutor, solutions or me, you are not ready. Please note that this does not mean you should *memorize* how to do the problems – as you know from the first exam, the exam will involve similar but not identical ideas. If you *understand* how to do all of these problems as well as all your past homework problems, and can use that understanding to *do* all the problems with no help, then you should be prepared.

• Topics:

- What a Golden Triangle is, what it has to do with φ and what it has to do with gnomons
- Fibonacci numbers
- How the Fibonacci numbers are related to φ
  - sequence of $\frac{F_n}{F_{n-1}}$
  - Binet's formula
  - anything else you can think of
- Using Binet's formula
- How/where the Golden Ratio shows up in a pentagon/pentagram.
- The distance formulae for points in 2-space and for points in 3-space
- Plotting points in 3-space
- The relationship between points in 3-space (as in all our cube problems)
- The Perspective Theorem-where it comes from, and using it
- The meaning of the word "orthogonal"

- Vanishing points - where do images of lines orthogonal to the picture plane vanish? How about lines parallel to the picture plane (the xy-plane)? Lines parallel to the "floor" (the xz-plane)? Lines parallel to a "side wall" (the yz-plane)?
- Vanishing points of parallel lines
- Finding the correct viewing position for a drawing in one-point perspective.
- The rules of perspective
- Subdividing rectangles into halves, fourths, eighths.
- Duplicating a rectangle immediately next to (attached to) your original.

Problem Set 8 Problems:

For these two problems (which are both either from or inspired by exercises from Lesson 4, *Lessons in Mathematics and Art*), print out a couple copies of the drawing of a section of a roadside fence (the link is with the link for this study guide)

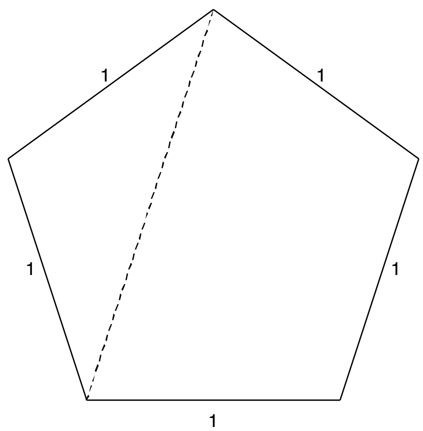
1. Within the solid outline of the fence section, draw 7 equally spaced vertical fenceposts to create a fence with 8 equal sections.
2. Extend the fence into the distance by drawing three exact perspective duplicates of the original rectangular section, each attached to the far side of the previous.

Study Guide Problems:

The following problems are intended as a supplement to your review; they are not intended to replace reviewing the reading and class notes, or redoing homework problems.

Remember: You are responsible for all material covered in your reading, whether or not we covered it in class.

1. The regular pentagon in the following figure has sides of length 1. Use the fact that the angle a diagonal forms with the closest side of the pentagon is 36°, along with the results of from some past homework problems, to show that the length of any one of its diagonals is φ.



2. Use that $F_{26} = 121,393$ and that $F_{28} = 317,811$, to find F_{29}.

3. Let a represent the 300th Fibonacci number and b represent the 301st Fibonacci number. Express the 298th Fibonacci number in terms of a and b. Simplify your answer.

4. Fact: $(F_1 + F_2 + F_3 + \ldots + F_N) + 1 = F_{N+2}$. Verify this fact for:

   (a) $N = 4$

   (b) $N = 10$

   Hint: Verify means show that the statement is true when $N = 4$, or $N = 10$. For instance, to verify that the statement is true for $N = 4$, you need to verify that the left side is equal to the right side. One approach would be to figure out what the left side *is*, and do the same thing for the right side. If they're equal, you have verified the statement.

5. *Calculating powers of* φ.

   Remember that φ is one of two solutions to $x^2 - x - 1 = 0$ $\left(\frac{1-\sqrt{5}}{2} \text{ is the other}\right)$. Of course, this means that $\varphi^2 - \varphi - 1 = 0$, or in other words, that $\varphi^2 = \varphi + 1$. *Check it out – compare* φ^2 *and* $\varphi + 1$ *on your calculator!*

   (a) Use that $\varphi^3 = \varphi^2 \cdot \varphi$, along with the above relationship, to show that $\varphi^3 = 2\varphi + 1$.

   (b) Use your result for φ^3 to show that $\varphi^4 = 3\varphi + 2$.

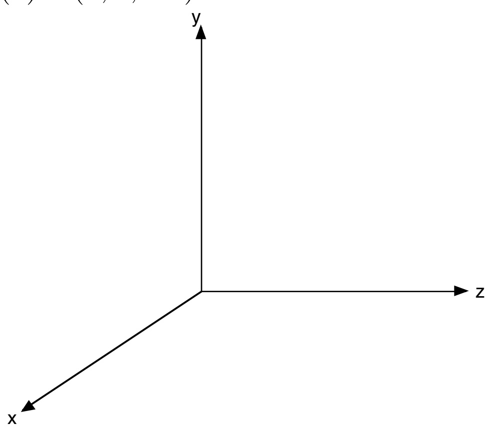
(c) Show that $\varphi^5 = 5\varphi + 3$.

(d) Look for a pattern in the results for φ^2, φ^3, φ^4, and φ^5. Based on what you see, what do you think φ^6 is ? Check your results.

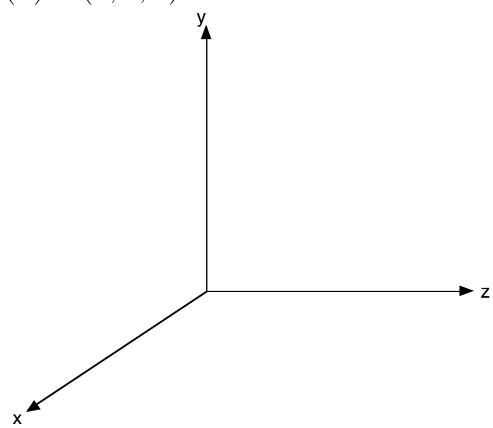
(e) In general, how do you think φ^N can be rewritten, in terms of just a single power of φ and some whole numbers?

6. Please plot the following points on a set of 3-D coordinate axes.

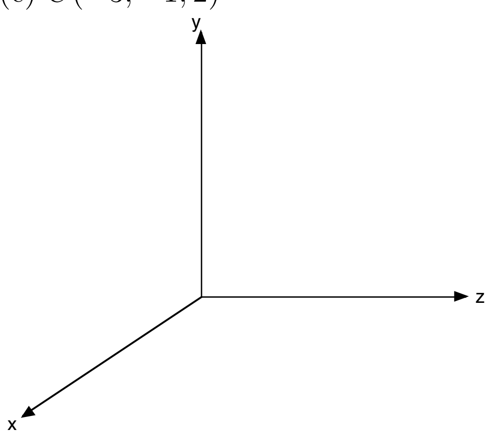
(a) $A(2, 0, -3)$



(b) $B(3, 1, 2)$



(c) $C(-3, -1, 2)$



7. Suppose we have a cube whose faces are again parallel to the coordinate planes, but only the coordinates of one corner are known. Suppose the corner that to the viewer appears to be the bottom left front corner has coordinates $(1, -3, 2)$ and that the length of each edge is 7.

(a) What are the coordinates of the other seven corners of the cube?

(b) Use the Perspective Theorem to find the perspective image of each of the eight corners. Use a viewing distance of 2 units.

(c) Draw the cube in perspective, using the images of each coordinate that you found in the previous part.

8. From the course website, print out a copy of Jan Van Eyck's *Giovanni Arnolfini and his Wife, Giovanna Cenami*, aka *The Arnolfini Wedding Portrait.* Mathematically analyze the perspective. Specifically: How can you tell, based on the evidence in the picture, that it was painted in one-point perspective (whether rigorously or not)? That is, how do you decide there appear to be lines representing orthogonals in the painting? Once you figure that out, investigate whether it was rigorously drawn in one-point perspective.

9. From the course website, print out a copy of Piero della Francesca's *The Flagellation.* On it,

   (a) Locate the primary vanishing point.

   (b) If there are any secondary vanishing points, find one.

   (c) Determine the ideal viewing position.

10. From the course website, print out a copy of Fra Francesco Colonna's *Garden of Love* from his *Hypnerotomachia Poliphili.* (It's small, do the best you can with it.)

    (a) Locate the primary vanishing point (as closely as you can).

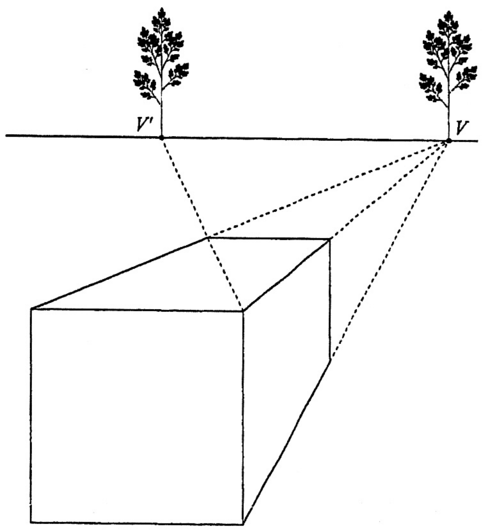
    (b) The two benches on the central platform each have vertical rectangular sides that are parallel to side walls. Assuming they are the same size, their diagonals should be parallel in real life, and hence the images of the diagonals should have the same vanishing point. Where should that vanishing point lie? Check it out – does it (or is it at least close?)

    (c) Similarly, the lines forming the diamonds on the side fences should form two sets of parallel lines parallel to side walls. What *should* be true about their vanishing points? Again, investigate whether it *is* true.

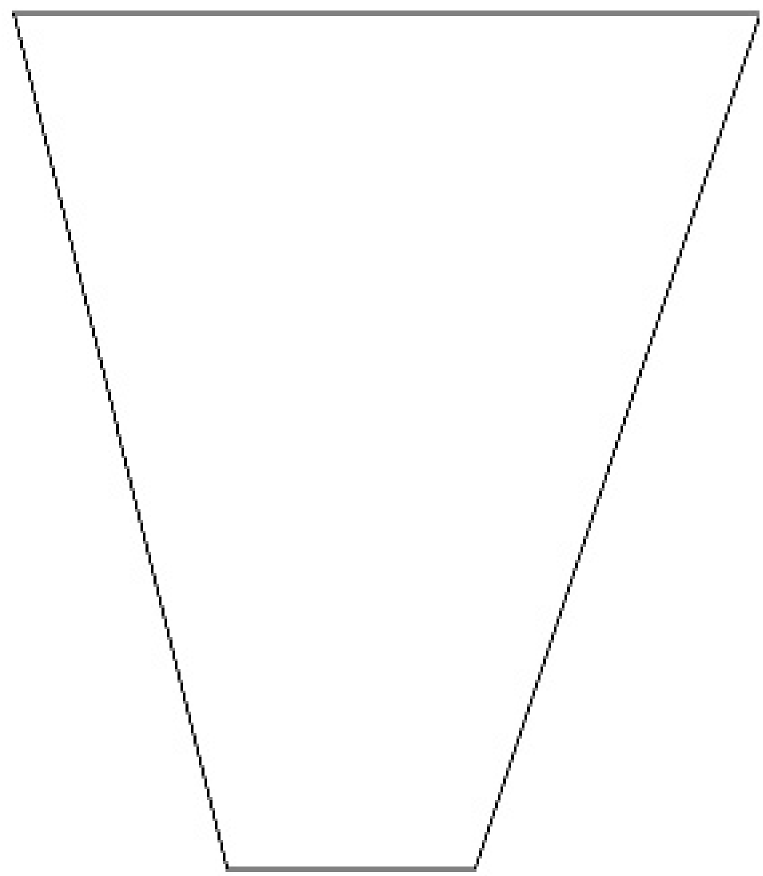
11. Draw a section of tile wall so that

    - the wall is a side wall extending orthogonally away from the picture plane, *and*
    - the tiles are all square, and the same size, *and*
    - the section of wall that you draw is 5 tiles deep and 5 tiles high, *and*
    - tiles are oriented with front & back edges parallel to the picture plane, while top & bottom edges are orthogonal, *and*
    - The viewing distance is 8"

12. If the box below represents a cube, then we can use our usual techniques to find the correct viewing position. But suppose the box is *not* a cube. Suppose instead that for whatever reason we know that the *side* of the box is intended to be three times as deep (that is, from front to back) as it is *tall*. Using similar ideas to those from class and the reading, determine what the viewing distance in this case is (from scratch).



13. Divide the perspective drawing of a rectangle below in half lengthwise, without measuring. (That is, draw a line that cuts the lines which no longer appear parallel in half.) Then divide the nearer of your halves in half; the nearer of your quarters in half, and the nearer of your eighths in half. In the end, the rectangle should have one half, one fourth, one eighth and two sixteenths.



14. Beginning with the rectangle shown below, which represents one brick, draw a portion of a brick wall consisting of identically sized bricks (in real-life) that is 3 bricks wide and 4 bricks high. Remember that in order to do this so it really looks like a brick wall, the second row of bricks must be offset from the first row, so that the end of one brick divides the brick below it in half.