- Exam 3 will take place on Wednesday 4/27/11, and will cover through Problem Set 12 (which I'm just including at the beginning of this study guide, rather than having it be separate) – that is, it will cover through introductory ideas of Other Dimensions.
- You may begin the exam at 8:30 am, or at 9:30 am (or anytime in between) – but I will collect it at 10:25 from all except those who have previously given me an accommodations letter and those who have an 8:30 class. *If you have an 8:30 class, please remember to contact me again about arrangements.*
- The solutions to Problem Sets 9 - 11 will soon be on 2-hour reserve at the circulation desk in the library (along with Problem Sets 1-7). Solutions to this study guide should become available on reserve Monday (if not Saturday).
- I will **not** be giving you a formula sheet along with your exam this time. You should know: the formula used in determining whether a point is in the Mandelbrot set.
- Advice:
    - As usual, spread studying for this exam out over several days. you can take breaks; you absorb more information; if some calamity occurs on the day before the exam, you've already done a fair amount of studying; you can get plenty of sleep the night before the exam; etc
    - Skim the readings and the notes from class with an aim to both reminding yourself of the big ideas and of making connections between ideas. Your main focus, however, should be to *do* (not just read through) as great a variety of problems as possible. In addition to doing the problems I've included on this study guide, you'll also want to redo as many problems as you can from the problem sets covered on this exam. (Again remember how important actually *redoing* the problems is.)
    - While you're working on problems, focus on *why* the steps are what they are. Spare some of your thoughts for how different problems are connected, and why various steps make sense.
    - When doing a problem that you've done before, don't waste your time trying to remember how you did it before–often, memory proves to be false and can lead you astray. Just focus on doing what makes sense.
    - Most people get the best results if most studying is done on their own, particularly as it gets closer to the day of the exam.
    - How long should you study for this? By now, you probably have a better sense for this than I do. If what you've been doing has been working for you; keep it

up. If not, study more *and* come talk to me about how to get more out of your studying.

- Remember: If you can not do the problems from start to finish without getting help from friend, tutor, solutions or me, you are not ready. Please note that this does not mean you should *memorize* how to do the problems – as you know from the first two exams, the exam will involve similar but not identical ideas. If you *understand* how to do all of these problems as well as all your past homework problems, and can use that understanding to *do* all the problems with no help, then you should be prepared.

- TOPICS:
  - Remember from Exam 2 how to subdivide a rectangle drawn in perspective into halves, quarters, eighths, etc, and how to draw an attached copy of a rectangle drawn in perspective.
  - Subdividing rectangles into portions that are *not* powers of 2 – thirds, fifths, etc.
  - Duplicating a rectangle so that there's an arbitrary amount of separation between the two, including the possibility of overlap.
  - Anamorphic art – drawing a picture that appears distorted unless viewed from an extreme viewpoint. We focused on planar anamorphic art. Be sure you understand the ideas behind it, as well as how to *do* it. **I will not be including any new problems on this in this study guide, but you still need to know how to do it**. Be sure you know which grid you draw the undistorted picture on, and which grid you draw the final distorted version on.
  - Understand the idea of symmetry of scale
  - Understand how to recursively construct a model of a fractal using geometry, as we did with the Koch Snowflake, the Sierpinski Triangle, the Sierpinski Carpet, and the Mitsubishi Gasket.
  - Be able to do the first several steps of creating a new fractal using similar geometric/visual recursion steps to those used to the above-mentioned fractals.
  - Understand how we found the dimension of the Koch Snowflake, the Sierpinski Triangle, the Sierpinski Carpet, and the Mitsubishi Gasket.
  - Adding and multiplying complex numbers.
  - Representing a point (a, b) as a complex number and vice versa.
  - What a seed is; using recursion/iteration to find the Mandelbrot sequence for a given seed (whether it's real or complex); what it means for a sequence to be *escaping*, *attracted*, and *periodic*; deciding whether a point belongs in the Mandelbrot set or not; and the distinction between coloring a point black or not-black.

– What a Linelander would see when a 2 dimensional object passes through its space, what a Flatlander would see when a 3 dimensional object passes through its space, and what we would see when a 4 dimensional object passes through our space. (I would only ask this for the appropriately dimensioned analog of the cube or sphere). By *see*, I really mean *experience through a combination of touch and sight, if given enough time to move as much as the dimensional restrictions allow.*

Problem Set 12 Problems:

These problems are all either from or inspired by exercises from Chapter 6, Symmetry, Shape, and Space.

1. In *Flatland*, A. Square has a conversation with his grandson, reproduced below:

   > *I began to show the boy how a Point by moving through a length of three inches makes a Line of three inches, which may be represented by 3; and how a Line of three inches, moving parallel to itself through a length of three inches, makes a Square of three inches every way, which may be represented by* 3^2 *[square inches].*

   Generalize to give a geometric significance to the quantity 3^3, thus answering a question posed by the grandson.

2. What would A. Square observe (assuming he has both the time and the effrontery to walk around and perhaps even touch), if a cube passed through Flatland:

   (a) so that one pair of faces is parallel to the plane of Flatland?

   (b) so that one corner approaching Flatland before the rest? (This one is tricky – do the best with it you can!)

3. What would a 4-dimensional cube look like to us if it passed through our 3-dimensional space "flush" with our space? (I'm of course having trouble finding words to describe, since we don't have many words to describe hyperspace, but what I'm asking you to do is the analogy of part (a) in the previous problem: one pair of cubes is "parallel" to our space). Describe and draw pictures as necessary to make your point.

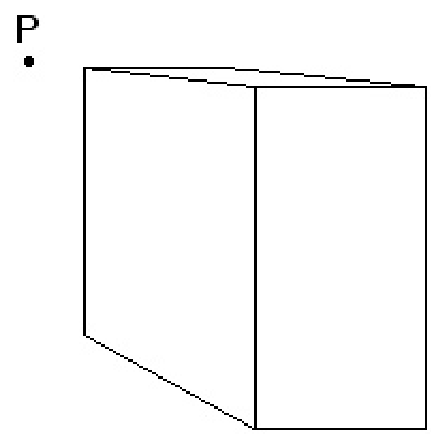
4. A circle is the set of all points in a plane equidistant from the center. A sphere is the set of all points in space equidistant from the center.

   (a) How would you define a fourth dimensional sphere, called a hypersphere?

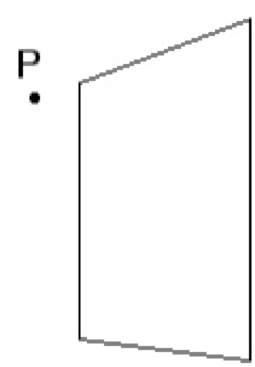
(b) What would we see if a hypersphere passed through our space? Determine this by thinking analogously – what did A. Square observe as the sphere passed through his space?

Study Guide Problems:

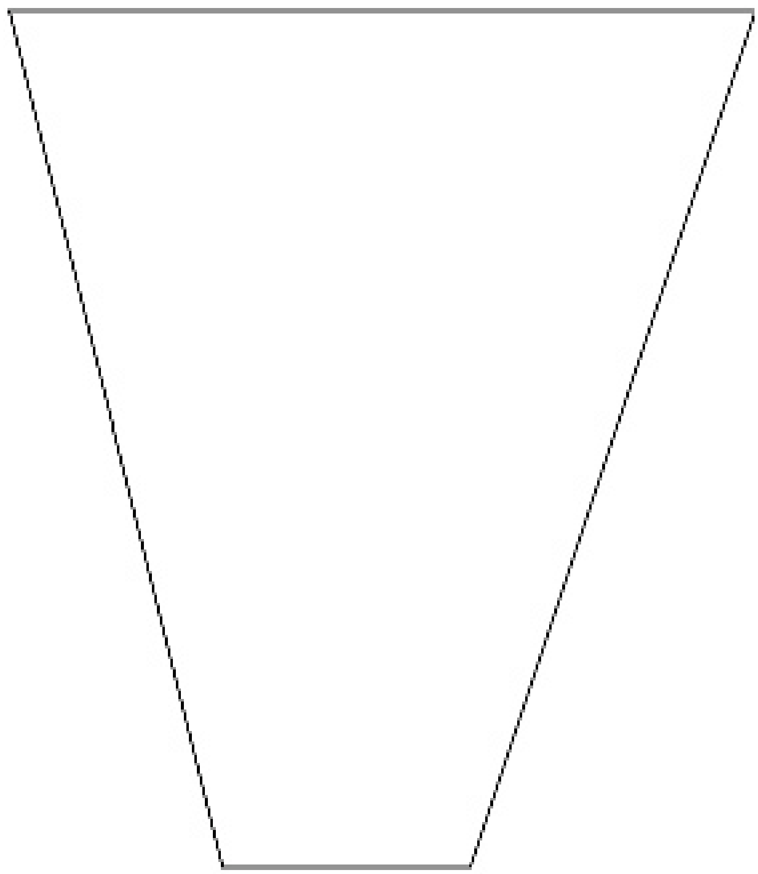
1. Below is a perspective drawing of a box, along with a point P. Draw a duplicate of the box, using the techniques we've developed. Place the duplicate so that it's front left corner (as we face it) is located at the point P, to create a picture of two boxes separated by some space.



2. Below is a perspective drawing of a window, retreating orthogonally to the picture plane. Draw a duplicate of this window, so that its upper rear corner is located at the point P, to create the appearance of a partially open sliding glass door.

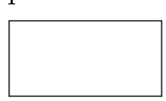
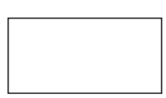
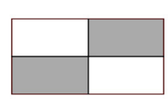


3. On the perspective drawing of a rectangle below, draw a horizontal line cutting the sides which no longer appear parallel into the division one-ninth/eight-ninths, without measuring. Probably the easiest way to do this is to divide the rectangle into thirds, and then one of the thirds on an end into thirds again.



4. *The Checkered Flag*

   (a) Using graph paper, carefully draw the figure that results after the first three steps, with the following recursive replacement rule:

   Start with a white rectangle . Whenever (and wherever) you see a white rectangle , replace it with a .

(b) Once again, the version you created above is actually the negative version of the actual Checkered Flag fractal, which would be black where yours is white and vice versa. Calculate the dimension of the *actual* Checkered Flag.

5. Evaluate the following, convert the results to the equivalent points, and graph them.

   (a) $3 - 5i + 7 - 11i$

   (b) $(4 + 7i) - (3 - 8i)$

   (c) $(2 - 3i)(2 + 3i)$

   (d) $(6 - 4i)^2$

6. For each of the following seeds s,

   (i) Find the first 5 terms of the Mandelbrot sequence with seed s

   (ii) Is this Mandelbrot sequence *escaping, periodic,* or *attracted*? (For some, you may not be sure; pick which you think is the most likely.)

   (iii) Will the point in the plane identified with the seed be a black point, or a non-black point?

   (a) $s = (0.75, 0)$

   (b) $s = (-0.75, 0)$

   (c) $s = (0, -2)$

   (d) $s = (-0.1, 0.1)$

7. What would A. Square observe (assuming he has the leisure to walk around and perhaps even touch), if a cube passed through Flatland edge first?