Most of these exercises are either from Chapter 6 in Symmetry, Shape, and Space*, or are inspired by that chapter.*

1. In *Flatland*, A. Square has a conversation with his grandson, reproduced below:

   > *I began to show the boy how a Point by moving through a length of three inches makes a Line of three inches, which may be represented by 3; and how a Line of three inches, moving parallel to itself through a length of three inches, makes a Square of three inches every way, which may be represented by* 3^2 *[square inches].*

   Generalize to give a geometric significance to the quantity 3^3, thus answering a question posed by the grandson.

2. What would A. Square observe (assuming he has both the time and the effrontery to walk around and perhaps even touch), if a cube passed through Flatland:

   (a) so that one pair of faces is parallel to the plane of Flatland?

   (b) so that one corner approaches Flatland before the rest? (This one is tricky – do the best with it you can!)

3. What would a 4-dimensional cube look like to us if it passed through our 3-dimensional space "flush" with our space? (I'm of course having trouble finding words to describe, since we don't have many words to describe hyperspace, but what I'm asking you to do is the analogy of part (a) in the previous problem: one pair of cubes is "parallel" to our space). Describe and draw pictures as necessary to make your point.

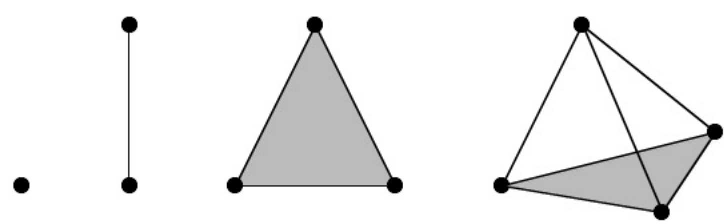
4. A circle is the set of all points in a plane equidistant from the center. A sphere is the set of all points in space equidistant from the center.

   (a) How would you define a fourth dimensional sphere, called a hypersphere?

   (b) What would we observe if a hypersphere passed through our space? Determine this by thinking analogously – what did A. Square observe as the sphere passed through his space?

5. Cubes and hypercubes are higher-dimensional analogues of the two-dimensional square. In Chapter 3 of *Dimensions*, (the chapter on the 4th dimension), you see the tetrahedron and the 4-dimensional analogue of the tetrahedron, both of which are higher-dimensional analogues of the two-dimensional triangle. On page 203-204 of *4th Dimension, Section 2* on OnCourse , your text discusses in detail how the three-dimensional tetrahedron is created, by moving from one dimension to the next as we did with the hypercube:

    > *Start with a single point. In the next generation, add another point above the first, and connect the two, creating a line segment. Place that line segment on the floor and add another point above the line segment and connect this point with each of the points on the line segment, obtaining a triangle. Place the triangle flat on the floor and add another point above it; connecting the new point to each of the points in the triangle gives a three-dimensional solid called the tetrahedron or triangular pyramid.*

    > *Adding another point in the fourth dimension,* ana *or* kata *the figure, and connecting this point to each point in the tetrahedron gives a figure called the* hypertetrahedron *or* pentahedroid.

    Using this description, develop formulae for the hypertetrahedron analogous to those we developed for the hypercube, and use them to fill in the following table. The thought process will be essentially identical to that we used for the hypercube, but of course since the figures you're working with are different, the formulae you develop will also be different .

Dimension	0D	1D	2D	3D	4D
Figure	point	segment	triangle	tetrahedron	hypertetrahedron
vertices v					
edges e					
faces f					
solids s					
4D regions t					