1. For each of the following, you'll be drawing a line that is cut in Extreme and Mean ratio (i.e. the Golden Ratio). Use what we have shown in class about $\frac{\text{whole}}{\text{greater}}$ and $\frac{\text{greater}}{\text{lesser}}$.

   (a) Suppose we want to draw a line cut in Extreme and Mean Ratio, and we want the short segment to be 3 cm long. How long should the long segment be? (Be exact, or round to 3 decimal places.) Draw such a line, using a ruler for exactness.

   (b) Suppose we want to draw a line of length 6 cm that is cut in Extreme and Mean ratio. Where should we place the cut? (Again, be exact or round to 3 decimal places.) Draw such a line, using a ruler for exactness.

   Look at the two lines with cuts that you've drawn. How are they similar?

2. Suppose we don't happen to agree with the ancient Greeks about balance between extremes. So we define our own standard of beauty and all that, beginning with the ideal way to cut a line. Here's ours:

   A line is said to be cut in the ***Very Cool Ratio*** *when the greater segment is to the lesser segment as* ***twice*** *the whole is to the greater.*

   (a) Set up the proportion described in the definition of the Very Cool Ratio.

   (b) What *is* this Very Cool Ratio? (By this I mean, *find* the number it equals.) Give it exactly (no decimals).

   (c) Suppose we want to draw a line cut in the Very Cool Ratio, and we want the short segment to be 3 cm long. How long should the long segment be? (Be exact or round to 3 decimals.) Draw such a line, using a ruler for exactness.

   (d) Suppose we want to draw a line of length 6 cm that is cut in the Very Cool ratio. Where should we place the cut? (Again, be exact or round to 3 decimal places). Draw such a line, using a ruler for exactness.

(e) How do the appearance of these cuts compare to those you found in the previous problem?

3. You want to paint a picture on a rectangular canvas, and you want the ratio of the length of the long side to the length of the short side to be φ, the Golden Ratio – that is, you want the canvas to be in the shape of a *Golden Rectangle*. If the short side of your canvas is 2' wide, exactly how long should you (try to) make the longer side be?

4. Wikipedia says that Salvador Dali "explicitly used the golden ratio in his masterpiece, *The Sacrament of the Last Supper* [shown below]. The dimensions of the canvas are a golden rectangle. " This painting is 267 cm wide and 166.7 cm tall.

(a) Find an acceptance range for the Golden Ratio, assuming the measurements are each accurate to within 0.5%. (You may assume that accuracy in measurements within 0.5% translates to accuracy in the ratio within 1%.)

(b) Based on this margin of error and the resulting acceptance range found in part (a), do the measurements support this claim?

(c) What do you conclude from this?

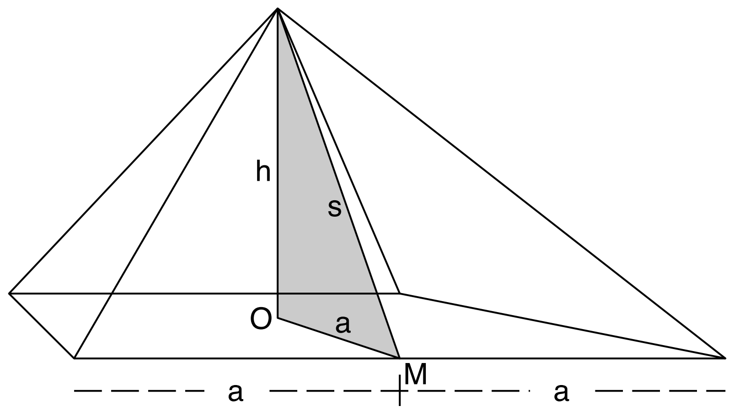
Note: You have read, or will read, that pentagons give rise to the Golden Ratio exactly. Notice that a huge dodecahedron (which is composed of pentagons) is suspended above and behind Jesus. Wikipedia also says that the edges of it appear in golden ratio to one another. Check it out if you like!

5. You have read, in Section 2.2.3, that it is frequently said that Herodotus described the construction of the Great Pyramid by saying that the Pyramid was built so that the area of each face would equal the area of a square whose side is equal to the Pyramid's height. You have also read that any pyramid (of any size) built following this relationship would ensure that the Golden Ratio appear in the Great Pyramid, but in the text I left out some details about *why* this is true.

   You will work through those details in this exercise, in the end showing that *any* pyramid built so that the area of each face equals the area of a square whose side is equal to the pyramid's height will have $\frac{\text{slant height}}{\text{half length of base}} = \varphi$. Since we have already shown in class that specifically for the Great Pyramid, $\frac{\text{slant height}}{\text{half length of base}}$ **is** well within any reasonable margin of error for φ, this will show that the Great Pyramid *was* built (whether intentionally or not, and whether or not Herodotus said so or not) so that the area of each face would equal the area of a square whose side is equal to the pyramid's height.

   Let

$$
\begin{aligned}
h &= \text{height of a pyramid} \\
a &= \frac{1}{2}(\text{length of that pyramid's base}) \text{ -- so } 2a = \text{length of base} \\
s &= \text{slant height} = \text{the height of that pyramid's triangular face}
\end{aligned}
$$



Remember: The following questions relate to any *pyramid built according to the statement attributed to Herodotus. That is, h, a, and s are all unknown lengths – don't use the specific dimensions for the Great Pyramid.*

(a) Find the area of a square whose sides all have length h (the height of the pyramid).

(b) Find the area of one of the triangular faces of the Pyramid.

Remember: Area of a triangle=$\frac{1}{2}$(base) $\times$ (height).

(c) Rewrite the statement attributed to Herodotus, using the expressions for area you found in parts 5a and 5b.

(d) By looking at the above diagram of the pyramid, find another equation that connects h, s, and a.

Hint: Look for a different triangle!

(e) Combine these two equations in a logical way to find a relationship between a and s. Solve for s/a. (You should get that $s/a = \varphi$! Depending on how you approach it, it may be helpful to remember that φ is the only positive real number $\square$ for which $\square^2 - \square - 1 = 0$, and it is also therefore the only positive real number $\square$ for which $\square = 1 + \frac{1}{\square}$.)

6. You have also read in Section 2.2.3 that if the Egyptians used rollers to measure the length of the base of the pyramid, and ropes to measure the height of the pyramid, then π would have been sure to appear in the Great Pyramid. I again left the details to this exercise.

   (a) Suppose you build a model of a pyramid as follows: take a wheel of diameter d and lay out a square base whose sides are each one revolution of the wheel long. Then make the pyramid height equal in length to two diameters of the wheel.

      i. How long is the base of your model? (Your answer will be in terms of d.)

         Remember: Circumference of a circle$= 2\pi\times$ *radius* $= \pi \times$ *diameter.*

      ii. How tall is your model? (Again, your answer will be in terms of d.)

      iii. Find the ratio of the height of your model to the length of the base of your model.

      iv. Find the ratio of the height of the Great Pyramid to the length of the side of the base of the Great Pyramid.

         Recall: The height of the Great Pyramid is 481.4 feet and the length of the side of the base of the Great Pyramid is 755.79 feet.

      v. Draw some conclusions about the shape of the Great Pyramid and the shape of your model.

   (b) In this part of the problem, you're going to show that the Egyptians wouldn't have had to use a gigantic measuring wheel for this process to have worked.

      i. Suppose you lay out a square base whose sides are each 10 revolutions of the wheel long, and you make the height be 20 diameters of the wheel. Find the ratio of the height of this new model to the base of this new model. How does it compare to the ratios you found in the previous parts?

      ii. Suppose you lay out a square base whose sides are each n revolutions of the wheel long, and you make the height be $2n$

diameters of the wheel tall. Again, find the ratio of the height of this new model to the base of this model, and compare.

(c) Using the dimensions for the Great Pyramid given above, find the diameter of the measuring wheel required so that 100 revolutions of the wheel would produce one side of the base of the Great Pyramid and 200 diameters would give the height. Is this a reasonable sized for the measuring wheel? That is, is it likely the Egyptians would use a measuring wheel this size, if they constructed the pyramid this way?