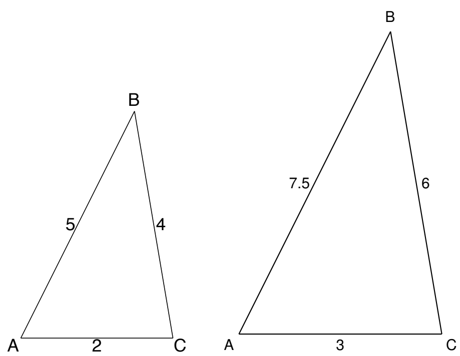
You may turn in your work on this handout if you find that easier, but it should still be neat and clear – not your rough draft.

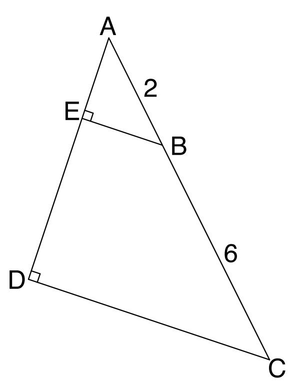
1. **Which are similar**

   (a) Which of the following pairs of figures are similar? If they are similar, explain why.

      i. the two triangles below:

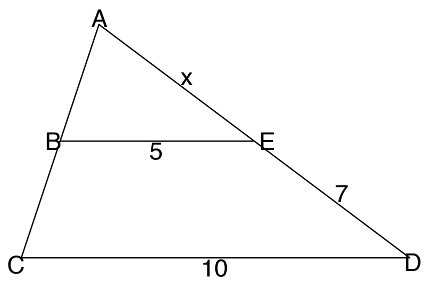


      ii. The pair below consists of the big triangle and the smaller one inside it.

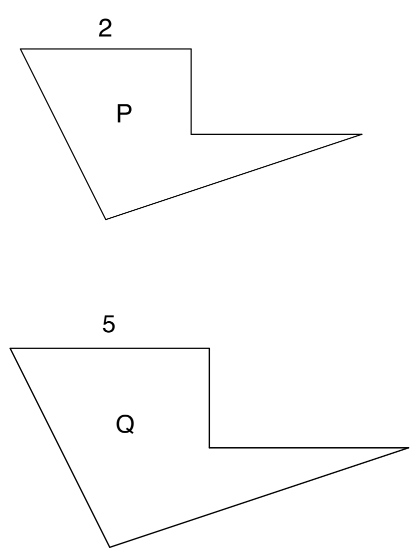


   (b) For the pair(s) above that you decided were similar, find the scale factor of the sides.

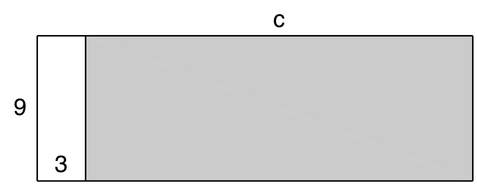
2. Assume that the following pair of triangles are similar, and find the unknown value x.



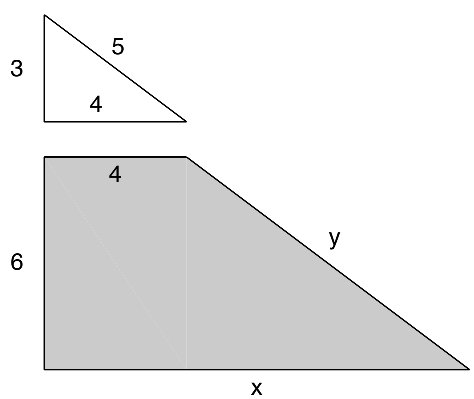
3. P and Q, shown below (but not to scale), are similar polygons. If the perimeter of P is 10, what is the perimeter of Q?



4. Find what c,which is the length of the shaded rectangle only, needs to be so that it is a gnomon to the white rectangle with sides 3 and 9.

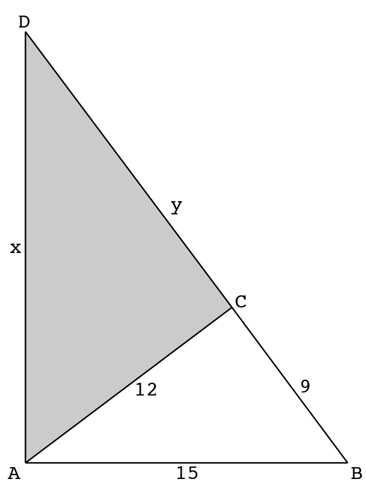


5. Find the values of x and y so that the shaded figure below is a gnomon to the white triangle.



6. Rectangle A is 10 by 20. Rectangle B is gnomon to rectangle A. What must the dimensions of rectangle B be?

7. Find the values of x and y so that the shaded triangle is a gnomon to the white triangle ABC.

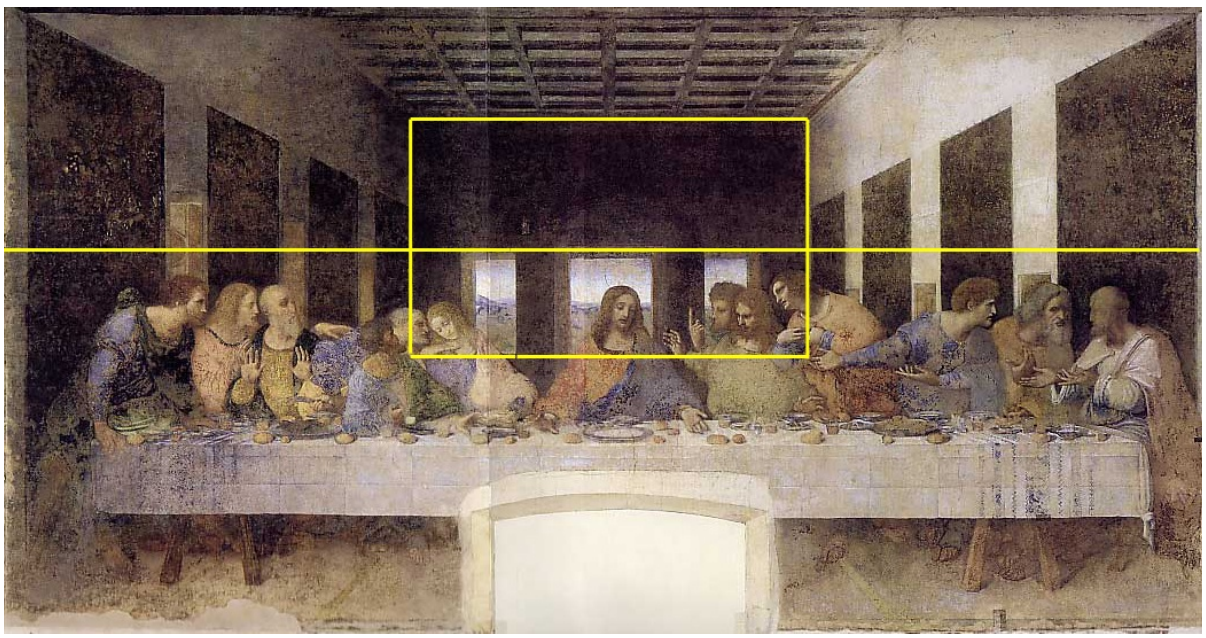


8. Rectangle A has a 10 by 10 square gnomon. What must be the dimensions of rectangle A?

9. In this problem (which spans several pages), you will be studying Leonardo da Vinci's *Last Supper*, and (ultimately) deciding whether you believe he was intentionally trying to incorporate the Golden Ratio. Measure as carefully as you can.

   (a) Find the acceptance range for φ based on a 2% margin of error in measurement (which you recall means you must allow for a 4% margin of error in your ratios).

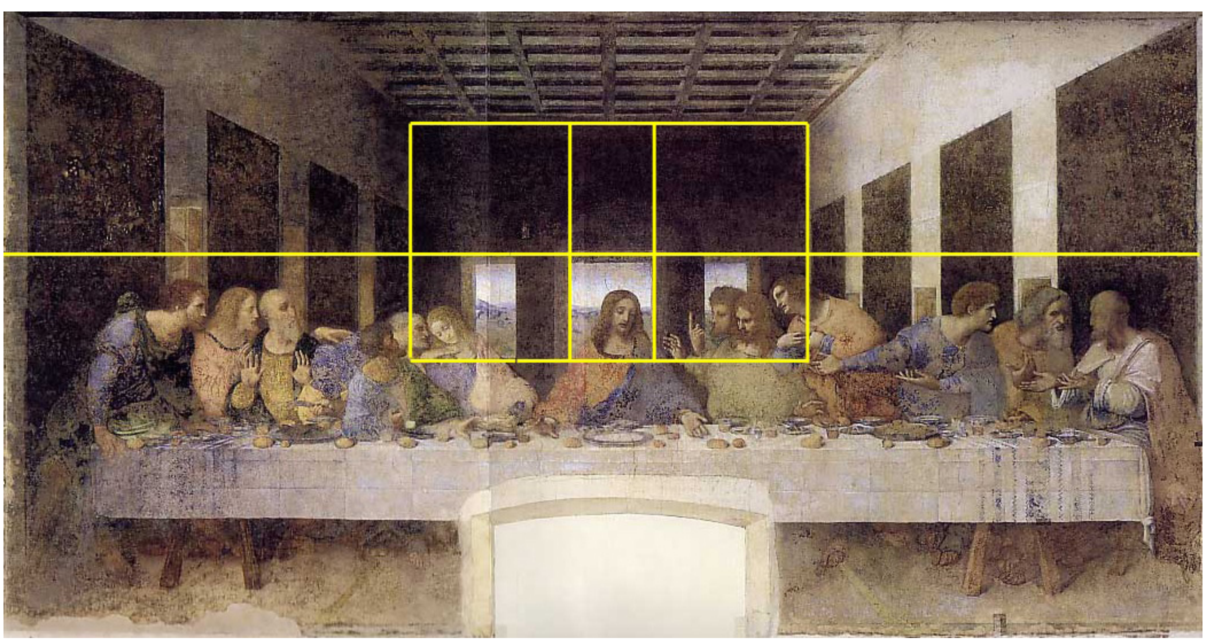
(b) Below, I have super-imposed a horizontal line that follows the tops of the insides of the windows on the rear wall all the way to the edges of the mural. Does that line cut the height of the painting into Extreme and Mean Ratio?



(c) In the figure above, I have also outlined the rectangle formed by the rear wall, down to the bottoms of the windows. Is it a Golden Rectangle (within our margin of error)?

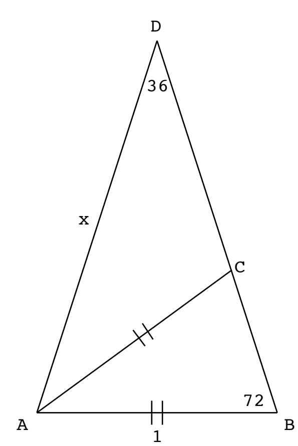
(d) Also in that figure, does the long horizontal line cut the height of the rear-wall rectangle in Extreme and Mean Ratio?

(e) Below, I have now super-imposed two vertical lines that upwardly extend the vertical lines that define the central window. These new vertical lines, along with the rectangle formed by the back wall, create rectangles to the left and right of the central window that are subdivided into a top part and a smaller rectangle below. Carefully measure the entire left rectangle and the entire right rectangle (that is, the rectangles that go from the top of the rear wall down to the bottom of the windows, and from the edges of the side walls to the window behind Jesus), and determine whether they are Golden, within our acceptance range.



(f) In the same figure, carefully measure both of the smaller rectangles (to the left and right of the central window), and determine whether they are Golden (within our margin of error).

10. *Golden Triangles:* In this exercise, you will (eventually) show that all of the triangles shown in the triangle below have the ratio *long side*:*short side* in the Golden Ratio. Since any triangles with the same angles will be similar, and hence have sides in the same proportions, this will show that any 72°-72°-36° triangle and any 36°-36°-108° triangle will be *Golden*, although you will of course first have to discover those angles. (Also, any isosceles triangle whose sides have this ratio will be similar to one of these two types.)



See next page for beginning of problem

(a) Show that $\triangle ABD$ is similar to $\triangle BCA$.
Hints:

- Since you're given that $\overline{AB} = \overline{AC}$, you know that $\triangle BCA$ is an isosceles triangle.
- Remember that the base angles of an isosceles triangle are equal, *and* if the base angles of a triangle are equal, it's an isosceles triangle.
- Although $\triangle ABD$ *looks* like an isosceles triangle, you are not given that it is – if you want to use that it is, you need to show it.
- Also remember that the sum of the angles in a triangle are 180°.

(b) Conclude that $\triangle ABD$ is an isosceles triangle

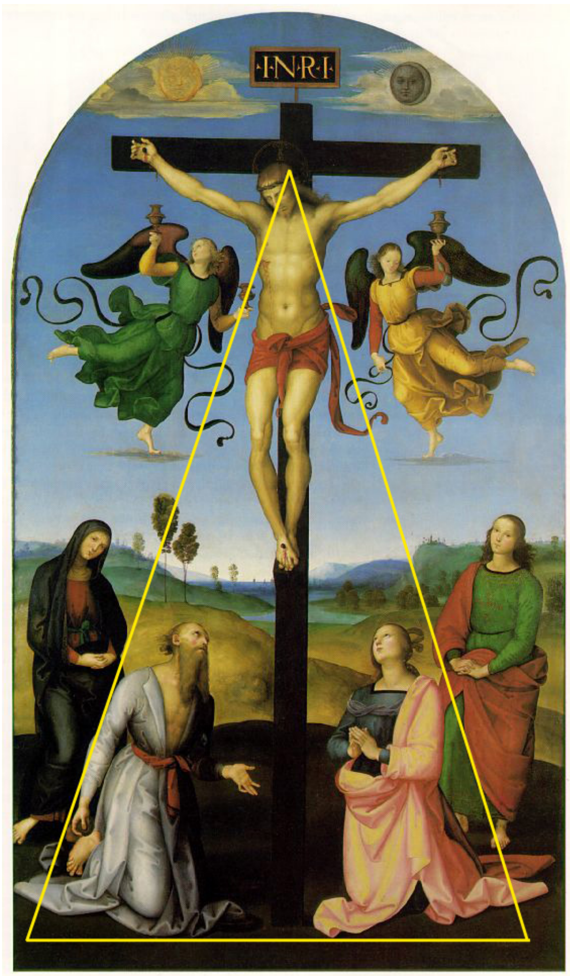
(c) Show that $\triangle ADC$ must also be an isosceles triangle.

(d) Use the result of part (b) to write the length $\overline{BC}$ in terms of x.

(e) Use the similarity you showed in (a), and your result to part (c), to show that $x = \varphi = \dfrac{1+\sqrt{5}}{2}$.

(f) Show that in the isosceles triangle $\triangle ADC$, the ratio of the longer side to the shorter side is again φ.

11. In this exercise, you will investigate the claim that Raphael's *Mond Crucifixion* exhibits a Golden Triangle – the triangle shown superimposed below.



(a) By carefully measuring this diagram, find the ratio of the length of the long side of this triangle to the base.

(b) Assuming a 2% margin of error in your measurements (as in Problem 2), does this triangle fall within the acceptance range for being a Golden Triangle?

(c) Do you think the placement of this triangle is defined in a natural or obvious way by the features of the painting, or do you think it was conveniently chosen to create a nearly golden triangle? In case the lines obscure some of the detail: the top vertex is at the point where the horizontal line defined by the bottom of the crossbar intersects the midline defined by Jesus' body; this point is also the rightmost edge of the crown of thorns. The left side of the triangle extends to follow the arm of St. Jerome (kneeling), while the right side extends to follow the cloak of John the Evangelist (standing). The horizontal leg just grazes the robe of St. Jerome on the left and the cloak of Mary Magdalene on the right.