*Because of all the figures, I made this like a worksheet, but you should still be turning in a detailed, complete, and neat argument, not just work, and not a rough draft.*

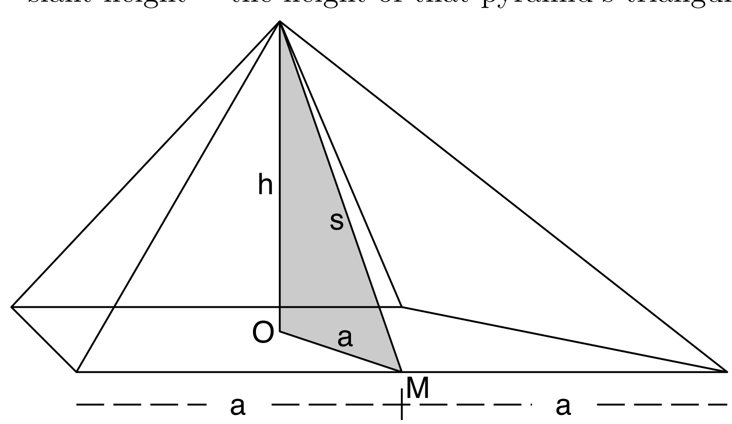
1. As you read in Section 2.2.3, it is frequently said that Herodotus wrote that the Great Pyramid was built so that the area of each face would equal the area of a square whose side is equal to the Pyramid's height.

   In this exercise, you will show that it is true that *any* pyramid built in such a way **will** have $\frac{\text{slant height}}{\text{half length of base}} = \varphi$.

   *(Since we have already shown in class that specifically for the Great Pyramid,* $\frac{\textit{slant height}}{\textit{half length of base}}$ **is** *well within any reasonable margin of error for* $\varphi$*, this will show that the Great Pyramid* WAS *built (whether intentionally or not, and whether or not Herodotus actually said so or not) so that the area of each face would equal the area of a square whose side is equal to the pyramid's height.)*

   Let

$$\begin{aligned} h &= \text{height of any pyramid} \\ a &= \frac{1}{2}(\text{length of that pyramid's base}) \\ 2a &= \text{length of pyramid's base} \\ s &= \text{slant height} = \text{the height of that pyramid's triangular face} \end{aligned}$$




   *The following questions relate to* any *pyramid built according to the statement attributed to Herodotus, so, $h$, $a$, and $s$ are all unknown lengths and should remain as variables. DO NOT use the specific measurements for the Great Pyramid of Giza.*

   (a) Find the area of a square whose sides all have length $h$ (the height of the pyramid).

(b) Find and simplify the area of one of the triangular faces of the Pyramid.

*Remember: Area of a triangle*=$\frac{1}{2}$*(base)* × *(height)*.

(c) Translate the statement attributed to Herodotus into a mathematical equation involving $a$, $s$, and $h$ using the expressions for area you found in parts 1a and 1b.

(d) By looking at the above diagram of a pyramid, find another equation that connects $h$, $s$, and $a$.

*Hint: Look for a different triangle!*

(e) Combine these two equations in a logical way to find a relationship between $a$ and $s$. Solve for $s/a$. (You should get that $s/a = \varphi$! Depending on how you approach it, it may be helpful to remember that $\varphi$ is the only positive real number $\square$ for which $\square^2 - \square - 1 = 0$, and it is also therefore the only positive real number $\square$ for which $\square = 1 + \frac{1}{\square}$.)

2. You have also read in Section 2.2.3 that if the Egyptians used rollers to measure the length of the base of the pyramid, and ropes to measure the height of the pyramid, then $\pi$ would have been sure to appear in the Great Pyramid. I again left the details to this exercise.

   (a) Suppose you build a model of a pyramid as follows: take a wheel of diameter $d$ and lay out a square base whose sides are each one revolution of the wheel long. Then make the pyramid height equal in length to two diameters of the wheel.

      i. How long is the base of your model? (Your answer will be in terms of $d$.)
      *Remember: Circumference of a circle*$= 2\pi\times$ *radius* $= \pi\times$*diameter.*

      ii. How tall is your model? (Again, your answer will be in terms of $d$.)

      iii. Find the ratio of the height of your model to the length of the base of your model.

      iv. Find the ratio of the height of the Great Pyramid to the length of the side of the base of the Great Pyramid.

**Recall:** The height of the Great Pyramid is 481.4 feet and the length of the side of the base of the Great Pyramid is 755.79 feet.

v. Draw some conclusions about the shape of the Great Pyramid and the shape of your model.

(b) In this part of the problem, you're going to show that the Egyptians wouldn't have had to use a gigantic measuring wheel for this process to have worked.

i. Suppose you lay out a square base whose sides are each 10 revolutions of the wheel long, and you make the height be 20 diameters of the wheel. Find the ratio of the height of this new model to the base of this new model. How does it compare to the ratios you found in the previous parts?

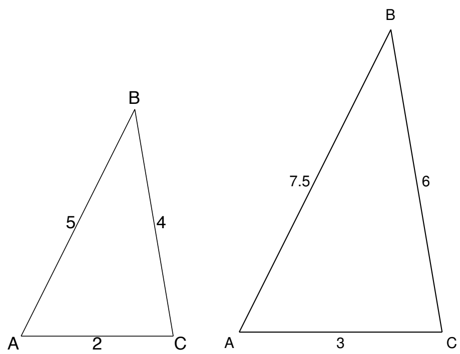
ii. Suppose you lay out a square base whose sides are each $n$ revolutions of the wheel long, and you make the height be $2n$ diameters of the wheel tall. Again, find the ratio of the height of this new model to the base of this model, and compare.

(c) Using the dimensions for the Great Pyramid given above, find the diameter of the measuring wheel required so that 100 revolutions of the wheel would produce one side of the base of the Great Pyramid and 200 diameters would give the height. Is this a reasonable sized for the measuring wheel? That is, is it likely the Egyptians would use a measuring wheel this size, if they constructed the pyramid this way?

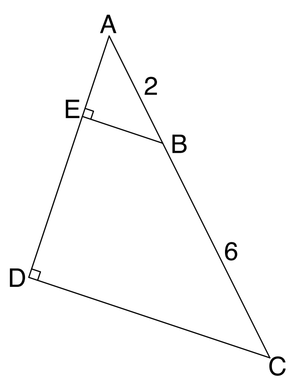
3. **Which are similar**

   (a) Which of the following pairs of figures are similar? If they are similar, explain why.

      i. the two triangles below:

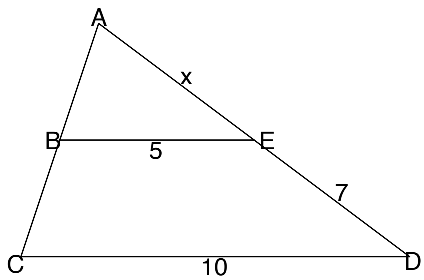



      ii. The pair below consists of the big triangle and the smaller one inside it.

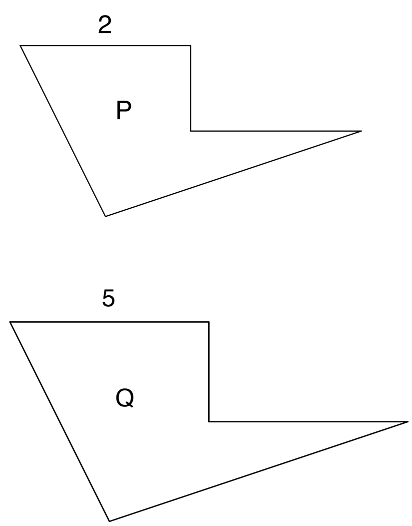



   (b) For the pair(s) above that you decided were similar, find the scale factor of the sides.

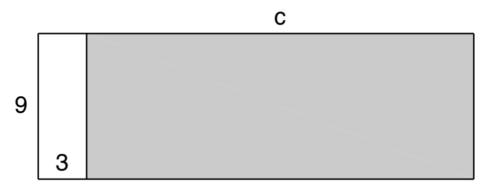
4. Assume that the following pair of triangles are similar, and find the unknown value $x$.




5. $P$ and $Q$, shown below (but not to scale), are similar polygons. If the perimeter of $P$ is 10, what is the perimeter of $Q$?



6. Find what $c$,which is the length of the shaded rectangle only, needs to be so that it is a gnomon to the white rectangle with sides 3 and 9.




7. Find the values of $x$ and $y$ so that the shaded figure below is a gnomon to the white triangle.