Some Sample Problems for Midterm 1

Midterm 1 will cover Sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 2.1, and 2.2.

You may have one sheet of notes. This should consist of no more than the front of a standard 8 1/2 x 11 sheet of paper. The notes should be handwritten by you, because the main reason I allow a sheet of notes is that I think the process of creating one is a valuable tool.

I write the exam intennding for it to take roughly 1 hour. However, since everyone works at different rates, I do allow for a bit of extra time. You have two options for when you will take the exam. The first option is from 12:30 -1:55 pm, Tuesday February 19, in our regular classroom. If you fear that 1 1/2 hours may not be enough time, then the second option is from 6:30-8:30 that evening, in room B243.

You probably don't need me to remind you of this, but you should plan on studying for this exam for a minimum of 8 hours, spread out over several days. Studying should consist not only of **deciding what to put on your cheat-sheet**, re-reading the text, class notes, and returned problem sets, but probably most importantly, of actually *doing* problems. This could consist of re-doing homework problems (without the solutions staring you in the face) or of choosing extra problems to do, or both. I have provided a few sample problems below, but ... the more problems you do, the better.

In short: The following problems are intended as a supplement to your review; they are not intended to replace reviewing the text and class notes, redoing homework problems, or choosing and doing extra problems from the text.

Another word of caution: You are responsible for all material covered in your reading, whether or not we covered it in class.

Before this exam, you should:

1. be very comfortable with functions, whether they are presented to you in the form of a formula, a table of data, a graph, or a description in words.

2. for a position function, know the connection between velocity, acceleration and the various derivatives.

3. know what the following are on a graph: a *root*, a *local maximum*, a *local minimum*, an *inflection point*, a *stationary point*, *increasing*, *decreasing*, *concave up*, *concave down*. You should also know the connections between these ideas and the first and second derivative, where appropriate.

4. know the effect of additive and multiplicative constants, inside and outside a function.

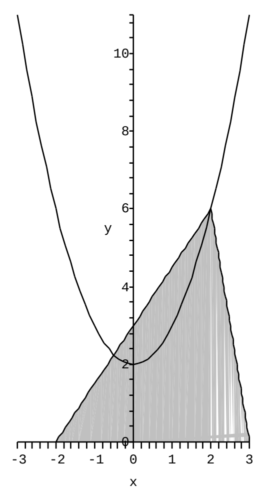
5. be able to tell whether a function is even or odd both graphically and algebraically.

6. be very comfortable with polynomials, sine, and cosine.

7. know the basic form and shape of an exponential function, and know which point is on the graph of every exponential function.

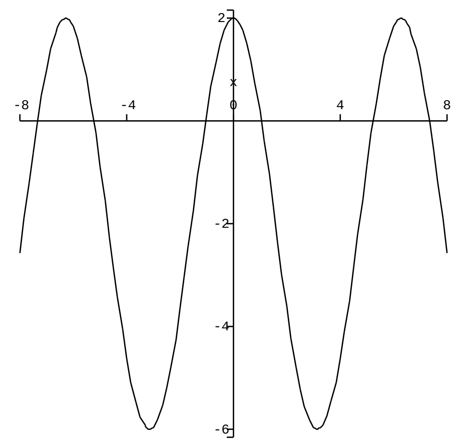
8. know the basic form and shape of a logarithmic function, and know which point is on the graph of every logarithmic function.
9. know the relationship between exponential and log functions.
10. know at least two interpretations of the derivative.
11. understand the idea of local linearity
12. know and be able to use the formal definition of the derivative
13. know what a difference quotient is – and know two interpretations of the difference quotient.
14. know how to find the derivative of a sum of powers of x without using the formal definition of the derivative.
15. You should be able to do all the problems below.

1. Find a formula for the area of the triangle shown below. The parabola is the graph of the function $f(x) = x^2 + 2$.



2. Let $f(x) = \sin(x)$. For all $x \neq 0$, define a function $g(x)$ as the slope of the line between $(0, 0)$ and $(x, f(x))$. Find an expression for $g(x)$.

3. Consider the graph shown below:



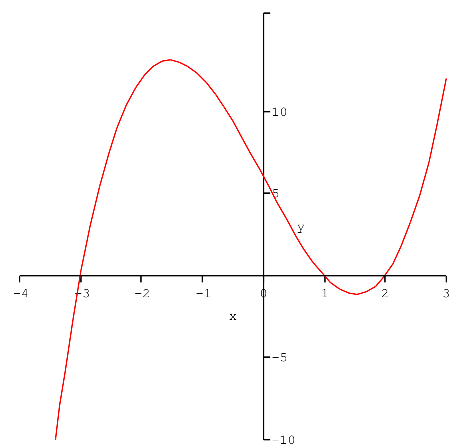
   (a) Find values A and D so that this is the graph of $y = A\cos(x) + D$.

   (b) Find values A, C, and D so that this is the graph of $y = A\sin(x + C) + D$.

4. Let f be a function whose derivative exists everywhere, and let $g(x) = f(x - 4) + 7$.

   Explain how the graphs of f and g are related. How are the graphs of f' and g' related?

5. Below is the graph of f'.



   (a) On which intervals is f increasing? decreasing?

   (b) Where does f have stationary points? Local maxima? Local minima?

   (c) Where is f concave up? concave down?

   (d) Where does f have inflection points?

   (e) Suppose that $f(0) = 1$. Sketch a graph of $y = f(x)$.

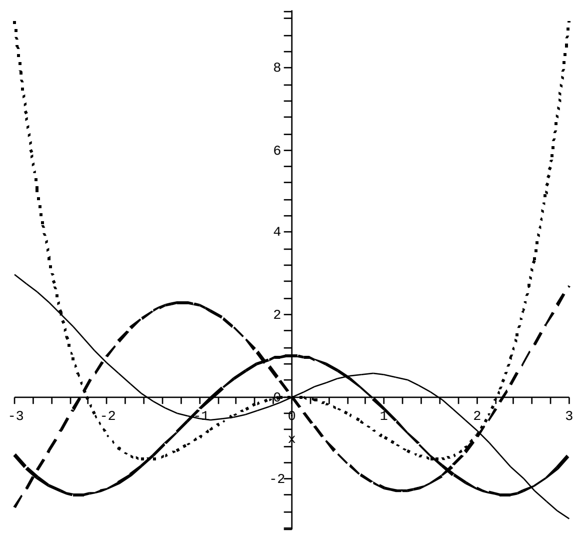
   (f) Suppose that $f(0) = 1$. Write the equation of the line tangent to the graph of $y = f(x)$ at $x = 0$.

6. Now suppose that the graph given in the previous problem is the graph of g''.

   (a) If $g'(1) = 50$, is g increasing or decreasing at $x = 2$? Why?

   (b) If $g'(0) = -1$, is g increasing or decreasing at $x = -1$? Why?

7. Graphs of f, f', f'', and an unrelated function g are shown below. Which is which? Justify your answer.



8. Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{3}{x^2}$.

9. Suppose that f is increasing and concave up on the interval $[0, 10]$ and that m is the slope of the secant line through the points $(3, f(3))$ and $(5, f(5))$.

   (a) Is $m < f'(3)$? Justify your answer with words and a picture.

   (b) Is $m < f'(5)$? Again, justify your answer.

10. Suppose that the average rate of change of a function h over the interval $[-3, 5]$ is 4. Is it possible that the line $y = 4x - 4$ passes through the points $(-3, h(-3))$ and $(5, h(5))$?

11. In Section 2.2, be able to do problems 17-24.

12. Let $f(x) = 2x^3 - x^2 - 8x + 7$. Find the *exact* values of the local maxima, local minima, and inflection points of $f(x)$.

13. Let $g(x) = 3x^4 - \dfrac{5}{x}$. Find the equation of the line tangent to $g(x)$ at $x = 1$.