Relationship btw antiderivatives & definite integrals:

1. The area function $\int_a^x f(t)\,dt$ is an antiderivative of $f(x)$
2. We use antiderivatives to find $\displaystyle\int_a^b f(x)\,dx$:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

In Class Work

Find the following indefinite integrals, and *check your answers!!*

1. $\int x \sin(\pi x^2)\, dx \qquad (u = \pi x^2)$
2. $\int \frac{1}{\sqrt{1-x}}\, dx \qquad (u = 1 - x)$
3. $\int \frac{x}{1+x^2}\, dx \qquad (u = 1 + x^2)$
4. $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\, dx \qquad (u = \sqrt{x+1})$

Solutions:

1. $\int x \sin(\pi x^2)\, dx \qquad (u = \pi x^2)$

- **Notice** u was chosen to be the *inside* of the composition.
- Identify u and find du:

$$u = \pi x^2 \qquad \Rightarrow u' = 2\pi x \qquad \Rightarrow du = 2\pi x\, dx.$$

Notice: $2\pi x\, dx$ isn't in integrand, but $x\, dx$ is

$$\Rightarrow x\, dx = \frac{1}{2\pi}\, du$$

- Replace appropriate expressions of x in the integrand with u and du:

$$\int x \sin(\pi x^2)\, dx = \int \sin(u) \cdot \frac{1}{2\pi}\, du.$$

Notice: there must be *no* terms involving x left

- Integrate the new simpler function:

$$\int \sin(u) \cdot \frac{1}{2\pi}\, du = \frac{1}{2\pi} \int \sin(u)\, du = \frac{1}{2\pi}(-\cos(u)) + C$$

Solutions:

2. $\displaystyle\int \frac{1}{\sqrt{1-x}}\,dx \qquad (u = 1-x)$

- **Notice** u was chosen to be the *inside* of the composition.
- Identify u and find du:

$$u = 1 - x \qquad \Rightarrow u' = -1 \qquad \Rightarrow du = -1\,dx.$$

Notice: $-1\,dx$ isn't in integrand, but dx is

$$\Rightarrow dx = -1\,du$$

- Replace appropriate expressions of x in the integrand with u and du:

$$\int \frac{1}{\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{u}} \cdot (-1)\,du.$$

Notice: there must be *no* terms involving x left

- Integrate the new simpler function:

$$\int \frac{1}{\sqrt{u}} \cdot (-1)\,du = -\int u^{-1/2}\,du = 2u^{1/2} + C$$

- Substitute back in $\int \frac{1}{\ldots}\,dx = 2\sqrt{1-x} + C$

Solutions:

3. $\int \frac{x}{1+x^2}\,dx \qquad (u = 1+x^2)$

- **Notice** The composition is not obvious (you can see it if you write $\frac{x}{1+x^2} = x(1+x^2)^{-1}$). When stuck, look for what portion of the integrand is particularly troublesome.
- Identify u and find du:

$$u = 1+x^2 \qquad \Rightarrow u' = 2x \qquad \Rightarrow du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$$

- Replace appropriate expressions of x in the integrand with u and du:

$$\int \frac{x}{1+x^2}\,dx = \int \frac{1}{2}\frac{1}{u}\cdot\,du.$$

- Integrate the new simpler function:

$$\frac{1}{2}\int \frac{1}{u} = \frac{1}{2}\ln|u| + C$$

- Substitute back in $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + C$

Solutions:

4. $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\, dx \qquad (u = \sqrt{x+1})$

- **Notice** Have a double-composition. Chose u to be biggest inside.
- Identify u and find du:

$$u = \sqrt{x+1} \Rightarrow u' = \frac{1}{2}(x+1)^{-1/2} \Rightarrow du = \frac{1}{2}\frac{1}{\sqrt{x+1}}\, dx \Rightarrow 2\, du = \frac{1}{\sqrt{x+1}}\, dx$$

- Replace appropriate expressions of x in the integrand with u and du:

$$\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\, dx = \int e^{\sqrt{x+1}}\frac{1}{\sqrt{x+1}}\, dx = \int e^u 2\, du$$

- Integrate the new simpler function:

$$2\int e^u = 2e^u + C$$

- Back substitute: $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\, dx = 2e^{\sqrt{x+1}} + C.$