In Class Work

Find the following indefinite integrals, and *check your answers!!*

1. $\int_0^{\pi/4} \sec^2 x \; e^{\tan x} \; dx$
2. $\int 12x \cos(3x^2 + 5) \; dx$
3. $\int_{-2}^{0} (x^2 + 4x)(x^3 + 6x^2 - 10)^{100} \; dx$
4. $\int \tan(x) \; dx$

   HINT: Write $\tan(x)$ as $\frac{\sin(x)}{\cos(x)}$
5. $\int_1^e \frac{1}{x \ln|x|} \; dx$
6. $\int x^2 \cos(x^3) \; dx$
7. $\int e^x \sqrt{e^x + 4} \; dx$
8. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} \; dx$
9. $\int x^2 e^{x^3+5} \; dx$
10. $\int \frac{\cos(1/x)}{x^2} \; dx$
11. $\int 2x \sec^2(1 - x^2) \; dx$
12. $\int \frac{x^2}{\sqrt{x+3}} \; dx$

Solutions

1. $\int_0^{\pi/4} \sec^2 x \, e^{\tan x} \, dx$

- Choose $u = \tan(x)$. Then $du = \sec^2(x)\, dx$
- Adjust limits:
  When $x = 0$, $u = \tan(0) = 0$; when $x = \pi/4$, $u = \tan(\pi/4) = 1$.
- Substitute:

$$\int_0^{\pi/4} \sec^2 x \, e^{\tan x} \, dx = \int_0^1 e^u \, du$$

- Integrate:

$$\int_0^{\pi/4} \sec^2 x \, e^{\tan x} \, dx = \int_0^1 e^u \, du = e^u \Big|_0^1$$

- Evaluate:

$$\int_0^{\pi/4} \sec^2 x \, e^{\tan x} \, dx = \int_0^1 e^u \, du = e^u \Big|_0^1 = e^1 - e^0 = e - 1$$

Solutions

2. $\int 12x\cos(3x^2+5)\,dx$

- Choose $u = 3x^2 + 5$. Then $du = 6x\,dx$
- Substitute:

$$\int 12x\cos(3x^2+5)\,dx = 2\int 6x\cos(3x^2+5)\,dx = 2\int \cos(u)\,du$$

- Integrate:

$$\int 12x\cos(3x^2+5)\,dx = 2\int \cos(u)\,du = 2\sin(u) + C$$

- Back-substitute:

$$\int 12x\cos(3x^2+5)\,dx = 2\int \cos(u)\,du = 2\sin(u)+C = 2\sin(3x^2+5)+C$$

Solutions

3. $\int_{-2}^{0} (x^2+4x)(x^3+6x^2-10)^{100}\, dx$

- Let $u = x^3+6x^2-10$. Then $du = (3x^2+12x)\, dx = 3(x^2+4x)\, dx$
- Adjust limits: $x=-2 \Rightarrow u = -8+24-10 = 6$; $x = 0 \Rightarrow u = -10$.
- Substitute:

$$\int_{-2}^{0} (x^2+4x)(x^3+6x^2-10)^{100}\, dx = \frac{1}{3}\int_{-2}^{0} 3(x^2+4x)(x^3+6x^2+4)^{100}$$

$$= \frac{1}{3}\int_{6}^{-10} u^{100}\, du$$

- Integrate:

$$\int_{-2}^{0} (x^2+4x)(x^3+6x^2+4)^{100}\, dx = \frac{1}{3}\int_{6}^{-10} u^{100}\, du = \frac{1}{3(101)} u^{101}\Big|_{6}^{-10}$$

- Evaluate:

$$\int_{-2}^{0} (x^2+4x)(x^3+6x^2+4)^{100}\, dx = \frac{1}{303}\left((-10)^{101} - 6^{101}\right)$$

Solutions

4. $\int \tan(x)\,dx$ HINT: Write $\tan(x)$ as $\frac{\sin(x)}{\cos(x)}$

- Composition isn't obvious, but once before, we let $u =$ denominator.
- Choose $u = \cos(x)$. Then $du = -\sin(x)\,dx$
- Substitute:

$$\int \tan(x)\,dx = -\int \frac{-\sin(x)}{\cos(x)}\,dx = -\int \frac{1}{u}\,du$$

- Integrate:

$$\int \tan(x)\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C$$

- Back-substitute:

$$\int \tan(x)\,dx = -\ln|u| + C = -\ln|\cos(x)| + C$$

Solutions

5. $\displaystyle\int_e^{e^e} \frac{1}{x \ln|x|}\, dx$

- Again have a fraction; this time, letting u be the entire denominator isn't going to work. Try letting u be just a part (the more complicated part) of the denominator.
- Choose $u = \ln|x|$. Then $du = \frac{1}{x}\, dx$
- Adjust limits: $x = e \Rightarrow u = 1 \qquad x = e^e \Rightarrow u = e$
- Substitute:

$$\int_e^{e^e} \frac{1}{x \ln|x|}\, dx = \int_e^{e^e} \frac{1}{x} \frac{1}{\ln|x|}\, dx = \int_1^e \frac{1}{u}\, du$$

- Integrate: $\displaystyle\int_e^{e^e} \tan(x)\, dx = \int_1^e \frac{1}{u}\, du = \ln|u| \Big|_1^e$
- Evaluate:

$$\int_e^{e^e} \frac{1}{x \ln|x|}\, dx = \ln|u| \Big|_1^e = \ln(e) - \ln(1) = 1$$

Solutions

6. $\int x^2 \cos(x^3)\, dx$

- Let $u =$ the inside of the composition $= x^3$
  Then $du = 3x^2\, dx$, so $\frac{1}{3}\, du = x^2\, dx$
- Substitute:
$$\int x^2 \cos(x^3)\, dx = \frac{1}{3} \int \cos(u)\, du$$
- Integrate:
$$\int x^2 \cos(x^3)\, dx = \frac{1}{3} \int \cos(u)\, du = \frac{1}{3} \sin(u) + C$$
- Back-substitute:
$$\int x^2 \cos(x^3)\, dx = \frac{1}{3} \int \cos(u)\, du = \frac{1}{3} \sin(u) + C = \frac{1}{3} \sin(x^3) + C$$

Solutions

7. $\int e^x \sqrt{e^x + 4}\, dx$

- Let $u =$ the inside of the composition $= e^x + 4$
  Then $du = e^x\, dx$
- Substitute:
$$\int e^x \sqrt{e^x + 4}\, dx = \int \sqrt{u}\, du$$
- Integrate:
$$\int e^x \sqrt{e^x + 4}\, dx = \int \sqrt{u}\, du = \frac{2}{3} u^{3/2} + C$$
- Back-substitute:
$$\int e^x \sqrt{e^x + 4}\, dx = \int \sqrt{u}\, du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (e^x + 4)^{3/2} + C$$

Solutions

8. $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

- Let $u =$ the inside of the composition $= \sqrt{x}$
  Then $du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2}\frac{1}{\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$
- Substitute:
$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = 2\int \sin(u)\,du$$
- Integrate:
$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = 2\int \sin(u)\,du = -2\cos(u) + C$$
- Back-substitute:
$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = 2\int \sin(u)\,du = -2\cos(u) + C = -2\cos(\sqrt{x}) + C$$

Solutions

9. $\int x^2 e^{x^3+5}\, dx$

- Let $u =$ the inside of the composition $= x^3 + 5$
  Then $du = 3x^2\, dx \Rightarrow \frac{1}{3}\, du = x^2\, dx$
- Substitute:
$$\int x^2 e^{x^3+5}\, dx = \frac{1}{3}\int e^u\, du$$
- Integrate:
$$\int x^2 e^{x^3+5}\, dx = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C$$
- Back-substitute:
$$\int x^2 e^{x^3+5}\, dx = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3+5} + C$$