Recall:

- We use the notation $\int_a^b f(x)\,dx$ to represent the signed area of between $f(x)$ and the x-axis from $x = a$ to $x = b$.

  We read this notation as *the definite integral of $f(x)$ from a to b*.

- $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

In Class Work

1. Let $f(x) = 2$ and $a = 0$. Then $A_f(x) = \int_0^x 2\,dt$.

   Using only information we've learned in this class (*no shortcuts!*),

   (a) Find $A_f(0)$, $A_f(3)$, $A_f(-3)$, and $A_f(4)$.

   (b) Based on the evidence you found in part (a), make a hypothesis about what a formula for $A_f(x)$ might be.

   (c) Show that your hypothesis is true for $x > 0$.

   (d) Show that your hypothesis is true for $x < 0$

   (e) How is your formula for $A_f(x)$ related to the original function $f(x) = 2$?

Solutions

1. Let $f(x) = 2$ and $a = 0$. Then $A_f(x) = \int_0^x 2\, dt$.
   (a) Find $A_f(0)$, $A_f(3)$, $A_f(-3)$, $A_f(4)$

$$
\begin{aligned}
A_f(0) &= \int_0^0 2\, dt = \textit{signed}\text{ area from 0 to 0 under line } y = 2 = 0 \\
A_f(3) &= \int_0^3 2\, dt = \textit{signed}\text{ area from 0 to 3 under line } y = 2 \\
&= \text{ area of rectangle of base 3 and height } 2 = 6 \\
A_f(-3) &= \int_0^{-3} 2\, dt = \textit{signed}\text{ area from 0 to -3 under horiz line 2} \\
&= -(\ \textit{signed}\text{ area from -3 to 0 under horiz line 2)} \\
&= -(\text{ area of rectangle with base 3 and height 2)} = -6 \\
A_f(4) &= \textit{signed}\text{ area from 0 to 4 under horiz line 2} \\
&= \text{ area of rectangle of base 4 and height } 22 = 8
\end{aligned}
$$

Solutions:

1. Let $f(x) = 2$ and $a = 0$. Then $A_f(x) = \int_0^x 2\ dt$.

   (b) Based on the evidence you found in part (a), you may have a hypothesis about what a formula for $A_f(x)$ might be.

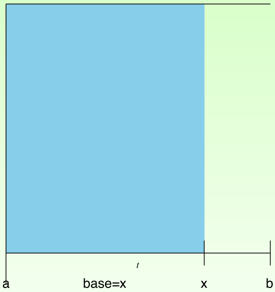
   Summarizing our results, we find that

x	$A_f(x)$
0	0
3	6
-3	-6
4	8

Hypothesis: It seems as if $A_f(x) = 2x$.

Solutions:

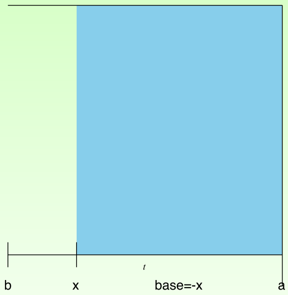
1. Let $f(x) = 2$ and $a = 0$. Then $A_f(x) = \int_0^x 2\ dt$.
   (c) Show that your hypothesis is true for $x > 0$.



We have a rectangle of base x and height 2, so $A_f(x) = 2x$.

Solutions:

1. Let $f(x) = 2$ and $a = 0$. Then $A_f(x) = \int_0^x 2\,dt$.
   (d) Show that your hypothesis is true for $x < 0$



For $x < 0$, we're going backwards.

$$A_f(x) = \int_0^x 2\,dt = -\int_x^0 2\,dt.$$

Now we have a rectangle of height 2 and base $-x$ (lengths must be positive).

$$A_f(x) = -2(-x) = 2x.$$

In either case (and also when $x = 0$), when $f(x) = 2$ and $a = 0$, $A_f(x) = 2x$.

In Class Work

2. Let $f(x) = 2x + 4$.
   (a) Use what we already know combined with high school geometry to find a formula for $A_{f,0}(x) = \int_0^x f(t)\,dt$.
   (b) Use (a), facts we've established in class about definite integrals, and more high school geometry, to find $A_{f,-2}(x) = \int_{-2}^x f(t)\,dt$
   (c) Use (a) or (b), facts we've established in class about definite integrals, and more high school to find $A_{f,1}(x) = \int_1^x f(t)\,dt$

Solutions

2 Let $f(x) = 2x + 4$.

(a) Use what we already know combined with high school geometry to find a formula for $A_{f,0}(x) = \int_0^x f(t)\, dt$.

$$\begin{aligned} \int_0^x 2t + 4\, dt &= 2\int_0^x t\, dt + 2\int_0^x 2\, dt \\ &= 2\left(\frac{1}{2}x^2\right) + 2(2x) = x^2 + 4x \end{aligned}$$

(b) Use (a), facts we've established in class about definite integrals, and more high school geometry, to find $A_{f,-2}(x) = \int_{-2}^x f(t)\, dt$

$$\begin{aligned} \int_{-2}^x 2t + 4\, dt &= \int_{-2}^0 2t + 4\, dt + \int_0^x 2t + 4\, dt \\ &= \text{area of a triangle of base 2 and height } 4 + (x^2 + 4x) \\ &= x^2 + 4x + 4 \end{aligned}$$

Solutions

2 Let $f(x) = 2x + 4$.

(c) Use (a) or (b), facts we've established in class about definite integrals, and more high school to find $A_{f,1}(x) = \int_1^x f(t)\,dt$

$$\begin{aligned}
\int_1^x 2t + 4\,dt &= \int_{-2}^x 2t + 4 - \int_{-2}^1 2t + 4\,dt \\
&= (x^2 + 4x + 4) - (\text{area of } \Delta \text{ of base 3 and height 6}) \\
&= (x^2 + 4x + 4) - 9 \\
&= x^2 + 4x - 5
\end{aligned}$$