# Recall:

- The definite integral $\int_a^b f(x)\,dx$ gives the signed area from $x = a$ to $x = b$ between $f(x)$ and the $x$-axis.
- We define the area function for a function $f(x)$, based at $x = a$, to be $A_f(x) = \int_a^x f(t)\,dt$, that is, the signed area from $t = a$ to $t = x$ between $f(t)$ and the $t$-axis.

## Recall:

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- We define the area function for a function $f(x)$, based at $x = a$, to be $A_f(x) = \int_a^x f(t)\,dt$, that is, the signed area from $t = a$ to $t = x$ between $f(t)$ and the $t$-axis.
- For several functions, all with basepoint of 0, we have found the corresponding area functions.

| $a$ | $f(x)$ | $A_f(x)$ |
|---|---|---|
| 0 | 1 | $x$ |
| 0 | 2 | $2x$ |
| 0 | $x$ | $\frac{1}{2}x^2$ |
| 0 | $x+2$ | $\frac{1}{2}x^2 + 2x$ |
| 0 | $2x+4$ | $x^2 + 4x$ |

# In Class Work from Thursday

2. Let $f(x) = 2x + 4$.

   (a) Use what we already know combined with high school geometry to find a formula for $A_{f,0}(x) = \int_0^x f(t)\,dt$.

   $$\int_0^x 2t + 4\,dt = 2\int_0^x t + 2\,dt = 2\left(\frac{1}{2}x^2 + 2x\right) = x^2 + 4x$$

   (b) Use (a), facts we've established in class about definite integrals, and more high school geometry, to find $A_{f,-2}(x) = \int_{-2}^x f(t)\,dt$

   (c) Use (a) or (b), facts we've established in class about definite integrals, and more high school to find $A_{f,1}(x) = \int_1^x f(t)\,dt$

# In Class Work

1. Suppose $f(x)$ is positive on some interval $[A, B]$ containing the base point $a$. What can you say about $A_f(x)$, the area function based at $t = a$, for all $x$ in $[A, B]$? What if $f$ is negative?

2. Suppose $f(x)$ is increasing on some interval $[A, B]$ containing the base point $a$. What can you say about $A_f(x)$, the area function based at $t = a$, for all $x$ in $[A, B]$? What if $f$ is decreasing?

# In Class Work

1. Suppose $f(x)$ is positive on some interval $[A, B]$ containing the base point $a$. What can you say about $A_f(x)$, the area function based at $t = a$, for all $x$ in $[A, B]$? What if $f$ is negative?

   Hint: Is $A_f(x)$ increasing or decreasing for all $x \in [A, B]$?

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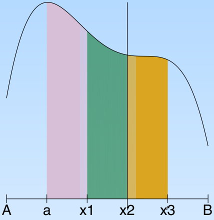
   Hint: Is $A_f(x)$ increasing or decreasing for all $x \in [A, B]$?

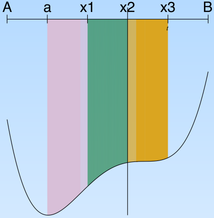
2. Suppose $f(x)$ is increasing on some interval $[A, B]$ containing the base point $a$. What can you say about $A_f(x)$, the area function based at $t = a$, for all $x$ in $[A, B]$? What if $f$ is decreasing?

   Hint: Is $A_f(x)$ concave up or concave down for all $x \in [A, B]$?

# Solutions

1. Suppose $f(x) \geq 0$ on $[A, B]$ containing $a$. What can you say about $A_f(x)$ based at $t = a$, for all $x$ in $[A, B]$? What if $f$ is negative?




$$A_f(x_1) \leq A_f(x_2) \leq A_f(x_3).$$

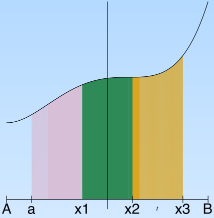
$f > 0 \Rightarrow$ as $x$ moves from left to right, $A_f(x) = \int_a^x$ includes more and more positive area, and so $A_f(x)$ is increasing.

$$A_f(x_3) \leq A_f(x_2) \leq A_f(x_1) \leq 0.$$

$f < 0 \Rightarrow$ as $x$ moves from left to right, $A_f(x) = \int_a^x$ includes more and more negative area, and so $A_f(x)$ is decreasing.

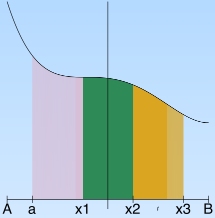
# Solutions

1. Suppose $f(x) \uparrow$ on $[A, B]$ containing $a$. What can you say about $A_f(x)$ based at $t = a$, for all $x$ in $[A, B]$? What if $f$ is $\downarrow$?




From $x_1$ to $x_2$, more area is added than from $a$ to $x_1$. Similarly, from $x_2$ to $x_3$, more area is added than from $x_1$ to $x_2$. Thus (in this case) $A_f(x)$ is increasing (since $f > 0$) at an increasing rate (since $f$ is increasing). In other words,

$A_f(x)$ is concave up.




From $x_1$ to $x_2$, less area is added than from $a$ to $x_1$. Similarly, from $x_2$ to $x_3$, less area is added than from $x_1$ to $x_2$. Thus (in this case) $A_f(x)$ is increasing (since $f > 0$) at a decreasing rate (since $f$ is decreasing). In other words,

$A_f(x)$ is concave down.

# Summary - Results for $A_f(x)$

$$A_f(x) \stackrel{def}{=} \int_a^x f(t)\,dt.$$

| $a$ | $f(x)$ | $A_f(x)$ |
|---|---|---|
| 0 | 1 | $x$ |
| 0 | 2 | $2x$ |
| 0 | $x$ | $\frac{1}{2}x^2$ |
| 0 | $x+2$ | $\frac{1}{2}x^2+2x$ |
| 0 | $2x+4$ | $x^2+4x$ |
| -2 | $2x+4$ | $x^2+4x+4$ |
| 1 | $2x+4$ | $x^2+4x-5$ |

| | |
|---|---|
| $f$ +/- | $A_f(x) \uparrow / \downarrow$ |
| $f \uparrow / \downarrow$ | $A_f(x) \smile / \frown$ |

Question: Does the relationship between $A_f(x)$ and $f(x)$ remind you of the relationship between $f(x)$ and any other function?