- **Theorem:** Suppose that $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$. Then, for any constants a and b, one antiderivative of $af(x) + bg(x)$ is $aF(x) + bG(x)$.

  In order to see that $aF(x) + bG(x)$ is an antiderivative of $af(x) + bg(x)$, we need to check that

  $$\frac{d}{dx}\big(aF(x) + bG(x)\big) = af(x) + bg(x).$$

  Since $F'(x) = f(x)$ and $G'(x) = g(x)$, then by combining Theorem 2.3.1 (i) and (iii), we see that:

  $$\frac{d}{dx}\big(aF(x)+bG(x)\big) = a\frac{d}{dx}\big(F(x)\big)+b\frac{d}{dx}\big(G(x)\big) = af(x)+bg(x).$$

- **Theorem:** For $x \neq 0$, $\frac{d}{dx}\big(\ln|x|\big) = \frac{1}{x}$.

  Recall that

  $$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0. \end{cases}$$

  If $x > 0$, then $\ln|x| = \ln(x)$, and so

  $$\frac{d}{dx}\big(\ln|x|\big) = \frac{d}{dx}\big(\ln(x)\big) = \frac{1}{x},$$

  while if $x < 0$, then $\ln|x| = \ln(-x)$, and so

  $$\frac{d}{dx}\big(\ln|x|\big) = \frac{d}{dx}\big(\ln(-x)\big) = (-1)\frac{1}{-x} = \frac{1}{x}.$$

  Thus in either case, $\frac{d}{dx}\big(\ln|x|\big) = \frac{1}{x}$.

- **Table of antiderivatives:**

  In each case, $F(x)$ represents *an* antiderivative of $f(x)$:

$f(x)$	$F(x)$	$f(x)$	$F(x)$
x^r, $r \neq 1$	$\dfrac{x^{r+1}}{r+1}$	$\sec(x)\tan(x)$	$\sec(x)$
$\sin(x)$	$\cos(x)$	$\csc(x)\cot(x)$	$-\csc(x)$
$\cos(x)$	$-\sin(x)$	e^x	e^x
$\sec^2(x)$	$\tan(x)$	$x^{-1} = \dfrac{1}{x}$, $x \neq 0$	$\ln\lvert x\rvert$
$\csc^2(x)$	$\cot(x)$		

- **Theorem:** Suppose that F and G are both antiderivatives of a function f on an interval I. Then

$$G(x) = F(x) + c,$$

  for some constant c.

  This is really just a restatement of Corollary 2.9.1, which states that if $g'(x) = f'(x)$ for all x in some open interval I, then for some constant c, $g(x) = f(x) + c$ for all $x \in I$.
  To see this, we have only to recall that since F and G are both antiderivatives of f on the interval I, it follows that on the interval I, $G'(x) = f(x) = F'(x)$.

  Thus by Corollary 2.9.1, there is some constant c such that $G(x) = F(x) + c$.