Rolle's Theorem:

Suppose that f is continuous on the interval $[a, b]$, differentiable on the interval (a, b), and $f(a) = f(b)$. Then there is a number $c \in (a, b)$ such that $f'(c) = 0$.

In Class Work

1. If $v(t) = e^{3t} + 7$, find *all* antiderivatives of $v(t)$.
2. A particle moves in a line with velocity $v(t) = e^{3t} + 7$. If the particle begins at position 0 (that is, if $p(0) = 0$), find the position function $p(t)$.

Solutions

1. If $v(t) = e^{3t} + 7$, find *all* antiderivatives of $v(t)$.

   One antiderivative: $\frac{1}{3}e^{3t} + 7t$

   Check: $\frac{d}{dt}\left(\frac{1}{3}e^{3t} + 7t\right) = \frac{1}{3}\left(3e^{3t}\right) + 7 = e^{3t} + 7.$

   All antiderivatives thus have the form

   $$\frac{1}{3}e^{3t} + 7t + C, \text{ where } C \text{ can be any constant}$$

2. A particle moves in a line with velocity $v(t) = e^{3t} + 7$. If the particle begins at position 0 (that is, if $p(0) = 0$), find the position function $p(t)$.

$$p(t) = \frac{1}{3}e^{3t} + 7t + C. \quad p(0) = 0 \implies 0 = \frac{1}{3}e^{0} + 0 + C$$

$$\implies C = -\frac{1}{3}$$

$$\implies p(t) = \frac{1}{3}e^{3t} + 7t - \frac{1}{3}$$