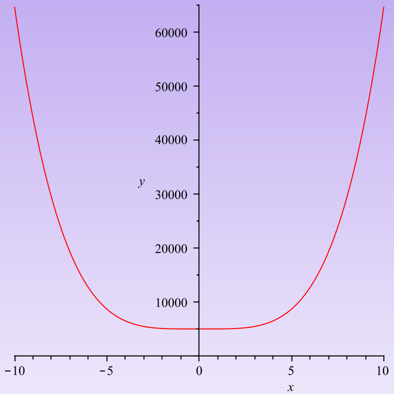
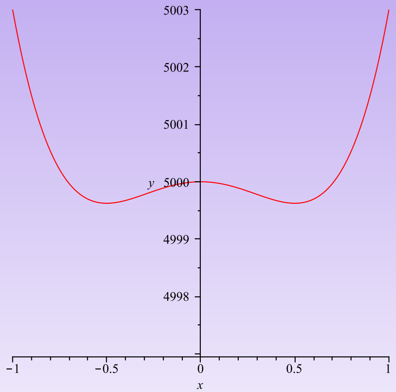
Windows matter!



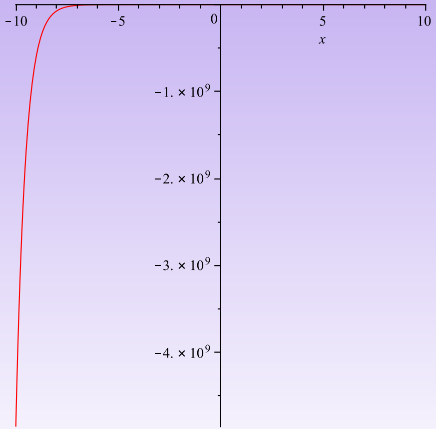
g on $[-10, 10]$



g on $[-1, 1]$

Example: Find all local extrema of $y = xe^{-2x}$.

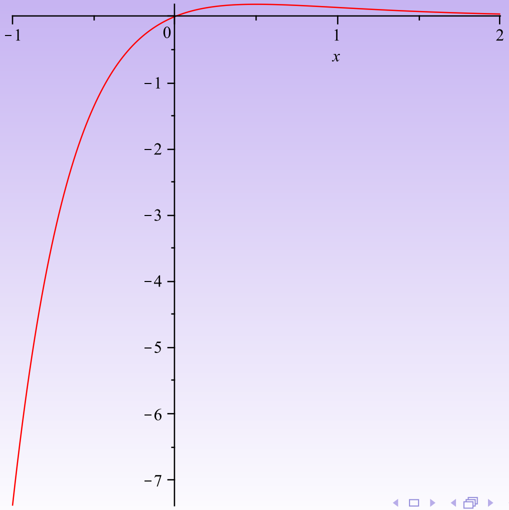
First, look at the graph using the standard graphing calculator interval of $[-10, 10]$:



Looks like it *might* be always increasing, becoming asymptotic to $y = 0$.

Example: Find all local extrema of $y = xe^{-2x}$.

Now that we know it has a local max at $x = 1/2$, we know roughly where to zoom in- try $x \in [-1, 2]$



In Class Work

1. Find (by hand) the intervals where the function is increasing and decreasing. Use this information (and a few key points) to sketch a graph. If you have access to graphing technology, then verify your results.
   (a) $y = x^3 - 3x + 2$
   (b) $y = (x+1)^{2/3}$
   (c) $y = \sin^2(x)$
2. Find (by hand) all critical numbers and use the First Derivative Test to classify each as the location of a local maximum, local minimum, or neither.
   (a) $y = x^4 + 4x^3 - 2$
   (b) $y = \sqrt{x^3 + 3x^2}$
   (c) $y = (x^2 + x + 0.45)e^{-2x}$

Solutions:

1. (a) $y = x^3 - 3x + 2$

- **Locate Critical Numbers**

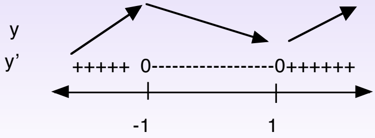
$$y' = 3x^2 - 3 \Rightarrow y' = 3(x^2 - 1) = 3(x-1)(x+1).$$

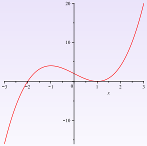
  - y' exists everywhere
  - $y' = 0$ at $x = -1$, $x = 1$.

  Thus $x = -1$ and $x = 1$ are the only two critical numbers.

- **Find where f is increasing, decreasing:**

$$f'(-2) = 3(-)(-) > 0 \quad f'(0) = 3(-)(+) < 0 \quad f'(2) = 3(+)(+) > 0$$





Solutions:

1. (b) $y = (x+1)^{2/3}$

- **Locate Critical Numbers:**

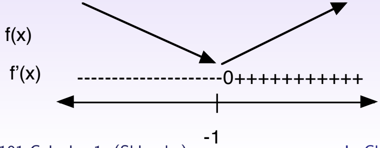
$$y' = \frac{2}{3}(x+1)^{-1/3} = \frac{2}{3(x+1)^{1/3}}.$$

  - y' does not exist at $x = -1$ (but y does)
  - $y' \neq 0$

  Thus $x = -1$ is the only critical number.

- **Find where f is increasing and decreasing:**

$$f'(-2) = \frac{2}{(+)(-)} < 0 \qquad f'(0) = \frac{2}{(+)(+)} > 0$$



Solutions:

1. (c) $y = \sin^2(x)$

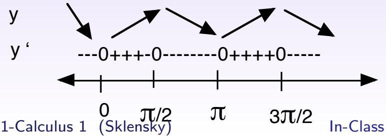
- **Locate Critical Numbers:**

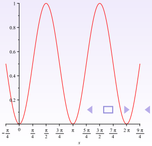
$$y' = 2\sin(x)\cos(x).$$

  - y' exists everywhere.
  - $y' = 0$ whenever either $\cos(x) = 0$ or $\sin(x) = 0$, so at $x = 0, \pi/2, \pi, 3\pi/2$ and also at all of these $\pm 2k\pi$.

- **Find where f is increasing and decreasing:**

$$f'(-\pi/4) = 2(-)(+) < 0, f'(\pi/4) = 2(+)(+) > 0,$$
$$f'(3\pi/4) = 2(+)(-) < 0, f'(5\pi/4) = 2(-)(-) > 0,$$
$$f'(7\pi/4) = 2(-)(+) < 0$$





Solutions:

2. (a) $y = x^4 + 4x^3 - 2$

- **Locate Critical Numbers:**

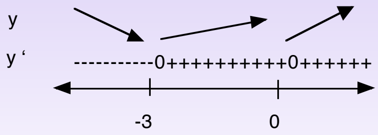
$$y' = 4x^3 + 12x^2 = 4x^2(x+3).$$

  - y' exists everywhere.
  - $y' = 0$ whenever either $x = 0$ or $x = -3$

  Thus only two critical points are $x = 0$ and $x = -3$

- **Find where f is increasing and decreasing:**

  $f'(-4) = 4(+)(-) < 0 \;\; f'(-2) = 2(+)(+) > 0 \;\; f'(1) = 2(+)(+) > 0$

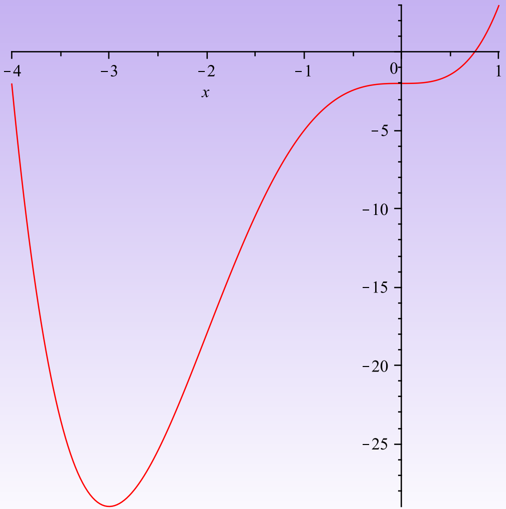


- **Conclude:** y has a local min at $x = -3$; $x = 0$ is neither a local min nor a local max.

Solutions:

2. (a)

Verify:



Solutions:

2. (b)$y = \sqrt{x^3 + 3x^2}$

- **Locate Critical Numbers:**

$$y' = \frac{1}{2}(x^3 + 3x^2)^{-1/2}(3x^2 + 6x) = \frac{3x^2 + 6x}{2\sqrt{x^3 + 3x^2}} = \frac{3x(x+2)}{2\sqrt{x^2(x+3)}}.$$

  - y' does not exist if: $x^2(x+3) < 0$ *or* if $x^2(x+3) = 0$ ... but if $x^2(x+3) < 0$, y does not exist either, so the only critical number from here are those where $x^2(x+3) = 0$, or $x = 0$ or $x = -3$.

    Note: $x^2(x+3) < 0$ *if* $x + 3 < 0$, *or if* $x < -3$, *so* y *only exists for* $x \geq -3$.

  - $y' = 0$ when the numerator is 0, so $x = -2$. (Not at $x = 0$, since that's not in the domain.)

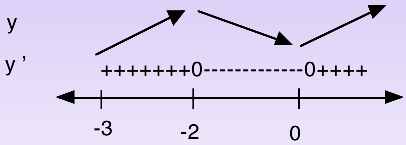
Thus our critical numbers are $x = -3$, $x = -2$ and $x = 0$.

Solutions:

2. (b)$y = \sqrt{x^3 + 3x^2}$

- **Critical Numbers:** $x = -3$, $x = -2$ and $x = 0$.
- **Find where f is increasing and decreasing:**
  Remember: y does not exist to the left of $x = -3$

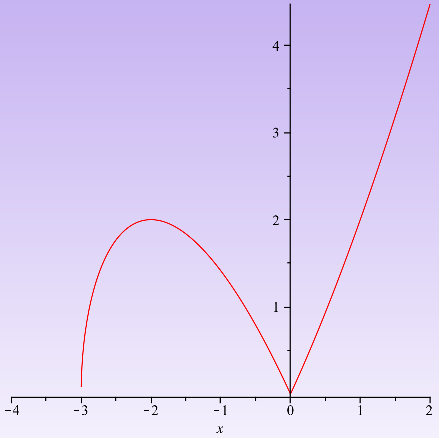
$$f'(-5/2) = \frac{(-)(-)}{+} > 0,\ f'(-1) = \frac{(-)(+)}{+} < 0,\ f'(1) = \frac{(+)(+)}{+} > 0$$



- **Conclude:**
  y has a local minimum at $x = -3$ and another at $x = 0$, and a local maximum at $x = -2$

Solutions:

2. (b) **Verify:**



Solutions:

2. (c) $y = (x^2 + x + 0.45)e^{-2x}$

- **Locate Critical Numbers:**

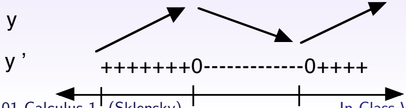
$$\begin{aligned} y' &= -2(x^2 + x + 0.45)e^{-2x} + (2x+1)e^{-2x} \\ &= e^{-2x}\big(-2x^2 - 2x - 0.9 + 2x + 1\big) \\ &= e^{-2x}\big(-2x^2 + 0.1\big) = \frac{-2(x^2 - 0.05)}{e^{2x}}. \end{aligned}$$

  - y' exists everywhere
  - $y' = 0$ when $x = \pm\sqrt{0.05}$

  Critical numbers: $x = -\sqrt{0.05} \approx -0.224$ and $x = \sqrt{0.05} \approx 0.224$.

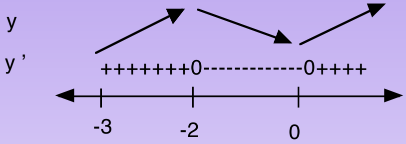
- **Find where f is increasing and decreasing:**

$$f'(-1) = \frac{-2(+)}{+} < 0 \quad f'(0) = \frac{-2(-)}{+} > 0 \quad f'(1) = \frac{-2(+)}{+} < 0$$



Solutions:

- **Find where f is increasing and decreasing:**



- **Conclude:**
  y has a local minimum at $x = -\sqrt{0.05}$ and a local maximum at $x = \sqrt{0.05}$
- **Verify:**