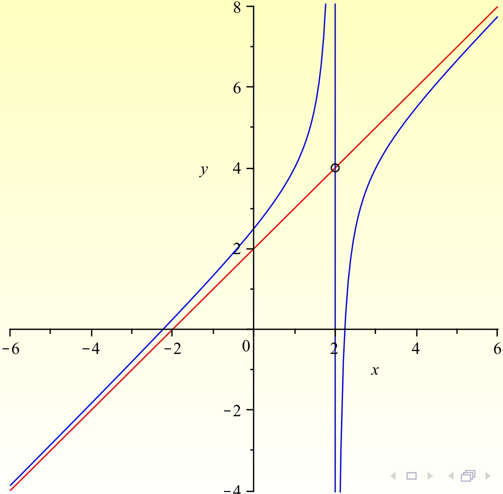
For Reading Question #1

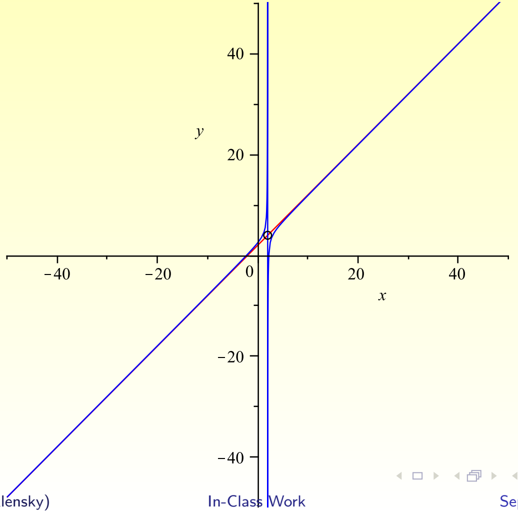
$$\frac{x^2-4}{x-2}, \frac{x^2-5}{x-2}$$



For Reading Question #1

Zooming out:

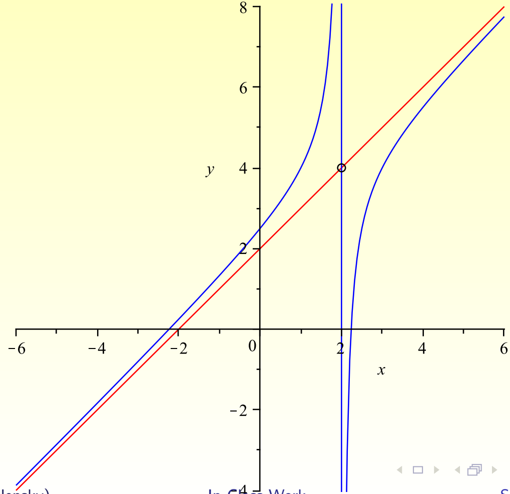
$$\color{red}{\frac{x^2-4}{x-2}}, \color{blue}{\frac{x^2-5}{x-2}}$$



For Reading Question #1

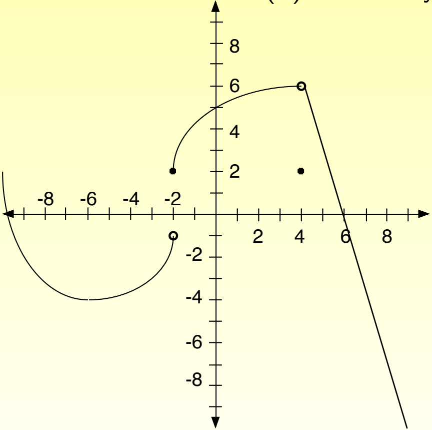
Zooming back in:

$$\textcolor{red}{\frac{x^2-4}{x-2}}, \textcolor{blue}{\frac{x^2-5}{x-2}}$$



In Class Work

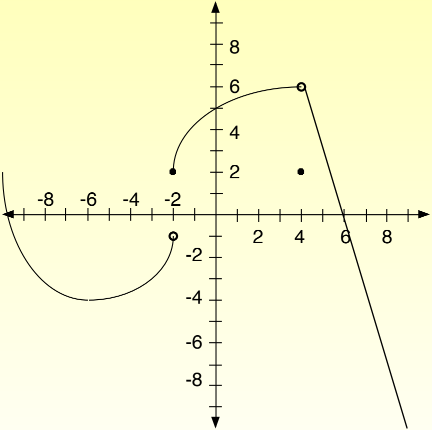
Consider the function $f(x)$ defined by the graph below. Find the following:



1. $f(4)$
2. $\lim_{x\to 4^+} f(x)$ and $\lim_{x\to 4^-} f(x)$
3. $\lim_{x\to 4} f(x)$
4. $f(-2)$
5. $\lim_{x\to -2^+} f(x)$ and $\lim_{x\to -2^-} f(x)$
6. $\lim_{x\to -2} f(x)$
7. $f(-6)$
8. $\lim_{x\to -6^+} f(x)$ and $\lim_{x\to -6^-} f(x)$
9. $\lim_{x\to -6} f(x)$

Solutions to In Class Work

Consider the function $f(x)$ defined by the graph below. Find the following:

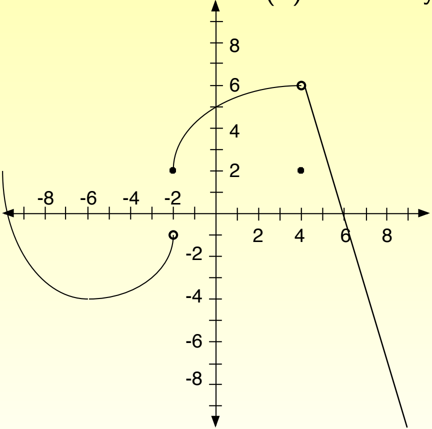


1. $f(4) = 2$:

   Above $x = 4$, we have both a closed and an open circle. The value of $f(4)$ is indicated by the y-value of the closed circle, so

Solutions to In Class Work

Consider the function $f(x)$ defined by the graph below. Find the following:



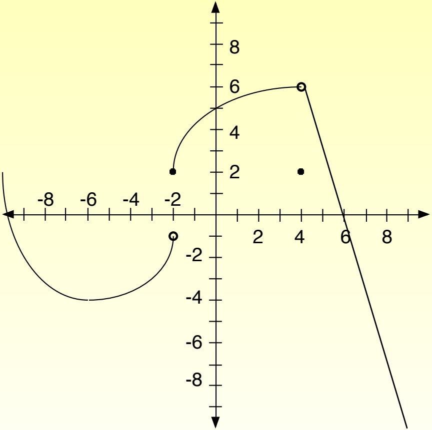
2. ▸ $\lim_{x \to 4^+} f(x) = 6$.

   To find the limit of $f(x)$ as x approaches 4 *from the right side*, choose a point on the curve to the right of $x = 4$ (but near $x = 4$), and travel along the curve toward $x = 4$. Observe that the y-values of the curve are getting closer and closer to 6 as x gets closer and closer to $x = 4$.

   Notice: for a limit, we do not pay any attention to what happens **at** $x = 4$.

Solutions to In Class Work

Consider the function $f(x)$ defined by the graph below. Find the following:

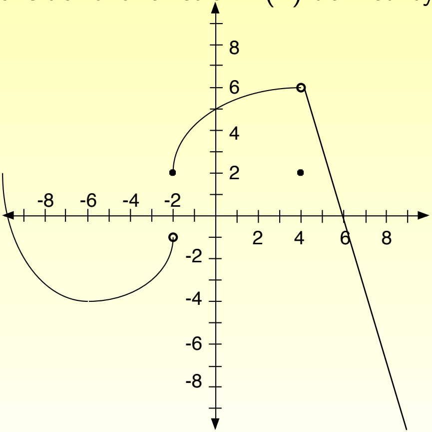


2. ▸ $\lim_{x \to 4^-} f(x) = 6$ also.

   To find the limit of $f(x)$ as $x \to 4$ from the left, choose a point on the curve to the *left* of $x = 4$ and move along the curve *toward* $x = 4$ (without ever reaching $x = 4$). Observe that the y-values are again getting closer and closer to 6.

Solutions to In Class Work

Consider the function $f(x)$ defined by the graph below. Find the following:



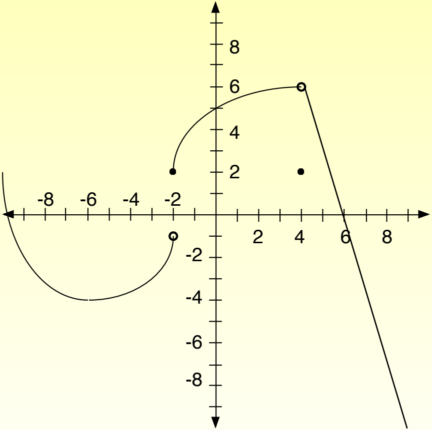
3. $\lim_{x \to 4} f(x) = 6$, since both the left- and right-limits were 6.

   Had they differed at all, we would say that the limit does not exist (*d.n.e.*).

   Notice: $f(4)$ did not equal $\lim_{x \to 4} f(x)$, and this is shown in the graph by the curve meeting at an empty circle, while the solid circle at $x = 4$ is at a different y-value.

Solutions to In Class Work

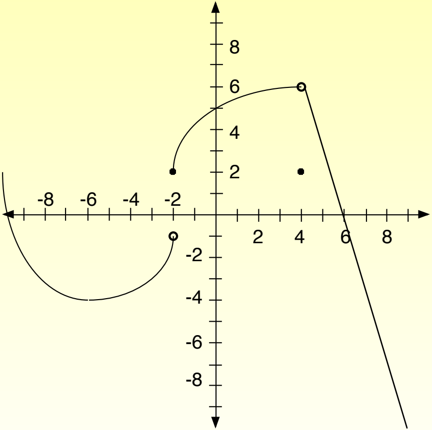
Consider the function $f(x)$ defined by the graph below. Find the following:



4. $f(-2) = 2$, because 2 is the y-value of the closed circle above $x = -2$.

Solutions to In Class Work

Consider the function $f(x)$ defined by the graph below. Find the following:



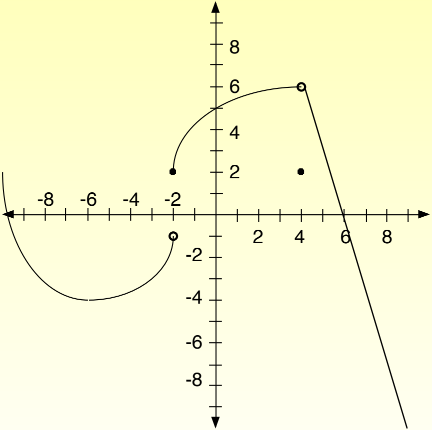
5.
- $\lim_{x \to -2^+} f(x) = 2$,

  because as we move along the curve toward $x = -2$ from the right (without ever reaching $x = -2$, the y-values approach $y = 2$.
- $\lim_{x \to -2^-} f(x) = -1$

  because as we move along the curve toward $x = -2$ from the left (without ever reaching $x = -2$, the y-values approach $y = -1$.

Solutions to In Class Work

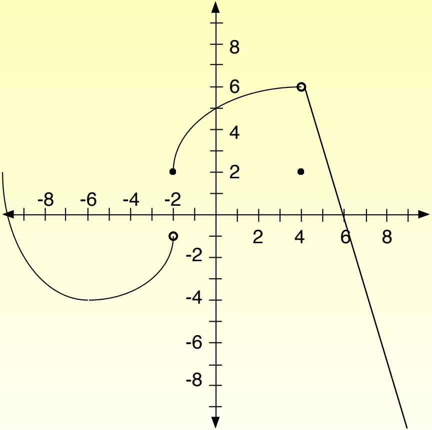
Consider the function $f(x)$ defined by the graph below. Find the following:



6. $\lim_{x \to -2} f(x)$ d.n.e. (does not exist), because the left- and right- sided limits differ.

Solutions to In Class Work

Consider the function $f(x)$ defined by the graph below. Find the following:



7. $f(-6) = -4$
8.
   - $\lim_{x \to -6^+} f(x) = -4$
   - $\lim_{x \to -6^-} f(x) = -4$
9. $\lim_{x \to -6} f(x) = -4$

Example 1.2.2, from text

Evaluate $\lim_{x \to -3} \frac{3x+9}{x^2-9}$.

$$\begin{aligned} \lim_{x \to -3^-} \frac{3x+9}{x^2-9} &= \lim_{x \to -3^-} \frac{3(x+3)}{(x+3)(x-3)} \\ &= \lim_{x \to -3^-} \frac{3}{x-3} = -\frac{1}{2} \end{aligned}$$

Cancel factors of $(x+3)$.

Example 1.2.2, from text

Evaluate $\lim\limits_{x \to -3} \dfrac{3x+9}{x^2-9}$.

$$\begin{aligned} \lim_{x \to -3^-} \frac{3x+9}{x^2-9} &= \lim_{x \to -3^-} \frac{3(x+3)}{(x+3)(x-3)} \\ &= \lim_{x \to -3^-} \frac{3}{x-3} = -\frac{1}{2} \end{aligned}$$

Cancel factors of $(x+3)$.

In the limit, the cancellation is legal, because x is never **equal to** -3.

Example 1.2.2, from text

Evaluate $\lim_{x\to -3} \dfrac{3x+9}{x^2-9}$.

$$\begin{aligned}\lim_{x\to -3^-} \frac{3x+9}{x^2-9} &= \lim_{x\to -3^-} \frac{3(x+3)}{(x+3)(x-3)} \\ &= \lim_{x\to -3^-} \frac{3}{x-3} = -\frac{1}{2}\end{aligned}$$

Cancel factors of $(x+3)$.

In the limit, the cancellation is legal, because x is never **equal to** -3.

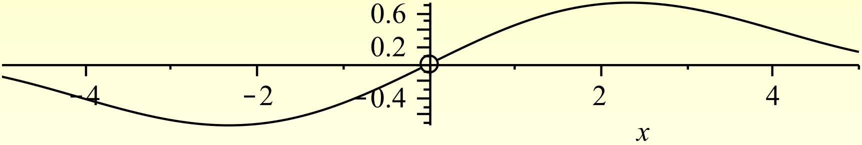
However, it is **not correct** to write

$$\frac{3(x+3)}{(x+3)(x-3)} = \frac{3}{x-3}$$

without some sort of note like when $x \neq -3$

To find the $\lim_{x\to 0} \frac{1-\cos(x)}{x}$:

1st Estimation: Look at a graph



To find the $\lim_{x\to 0} \dfrac{1-\cos(x)}{x}$:

2nd Estimation: Create a table of values of $f(x)$:

From the left:

x	$\dfrac{1-\cos(x)}{x}$
-0.1	-0.049958
-0.01	-0.005000
-0.001	-0.000500
-0.0001	-0.000050
-0.00001	-0.000005

From the right:

x	$\dfrac{1-\cos(x)}{x}$
0.1	0.049958
0.01	0.005000
0.001	0.000500
0.0001	0.000050
0.00001	0.000005