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- $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$, **if** $\lim\limits_{x\to a} g(x) \neq 0$

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- $\lim_{x\to a} p(x) = p(a)$,
- $\lim_{x\to a} \left(f(x)\right)^{1/n} = \left(\lim_{x\to a} f(x)\right)^{1/n}$, as long as the root makes sense.

Limits you will find helpful:

For any real number a, we have:

(i) $\lim_{x \to a} \sin(x) = \sin(a)$

(ii) $\lim_{x \to a} \cos(x) = \cos(a)$

(iii) $\lim_{x \to a} e^x = e^a$

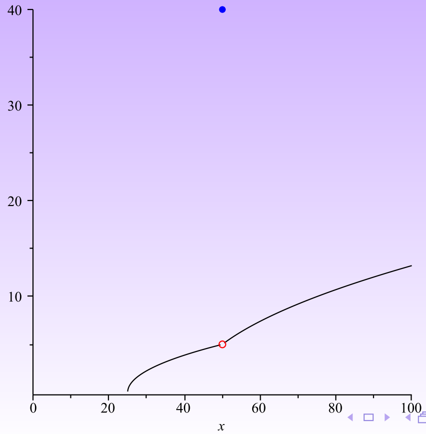
(iv) $\lim_{x \to a} \ln(x) = \ln(a)$, for $a > 0$

(v) If $p(x)$ is a polynomial, then $\lim_{x \to a} f\left(p(x)\right) = \lim_{x \to p(a)} f(x)$.

(vi) $\lim_{x \to 0} \frac{\sin(x)}{x} = 0$

Graph of function for Example:

$$f(x) = \begin{cases} \sqrt{x-25} & \text{if } x < 50 \\ 40 & \text{if } x = 50 \\ \sqrt{3x-125} & \text{if } x > 50 \end{cases}$$



In Class Work

1. $\lim_{x \to 0} (x^2 - 3x + 1)$
2. $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 4}$
3. $\lim_{x \to -1} f(x)$, where $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$
4. $\lim_{h \to 0} \dfrac{(2+h)^2 - 4}{h}$

Solutions:

1. $\lim_{x \to 0}(x^2 - 3x + 1)$

   Because $x^2 - 3x + 1$ is a polynomial, we know that

   $$\lim_{x \to a} p(x) = p(a),$$

   or in this case,

   $$\lim_{x \to 0}(x^2 - 3x + 1) = 0^2 - 3 \cdot 0 + 1 = 1.$$

Solutions:

2. $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 4}$

We know that $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$, if $\lim_{x \to a} g(x) \neq 0$.

Unfortunately, $\lim_{x \to 2}(x^2 - 4)$ is 0, so can't use that result.

Factor!

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 4} = \lim_{x \to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)}.$$

As we take the limit as x approaches 2, we're not ever letting x be 2, so cancelling the common terms is legal.

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 4} = \lim_{x \to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)} = \lim_{x \to 2} \frac{x+1}{x+2} = \frac{\lim_{x \to 2}(x+1)}{\lim_{x \to 2}(x+2)} = \frac{3}{4}.$$

Solutions:

3. $\lim_{x \to -1} f(x)$, where $f(x) = \begin{cases} 2x+1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$

   $\lim_{x \to -1^{-1}} f(x) = \lim_{x \to -1^-} 2x+1 = 2(-1)+1 = -1$, since $2x+1$ is a polynomial.

   $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 2x+1 = 2(-1)+1 = -1$, because $2x+1$ is still a polynomial.

   Since the left- and right-sided limits agree, $\lim_{x \to -1} f(x) = -1$ (even though $f(-1) = 3$).

Solutions:

1. $\lim_{h\to 0} \dfrac{(2+h)^2 - 4}{h}$

Can't use that $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{f(a)}{g(a)}$, since the denominator approaches 0.

Expand numerator, see if we can come up with anything!

$$\lim_{h\to 0} \frac{(2+h)^2 - 4}{h} = \lim_{h\to 0} \frac{4 + 4h + h^2 - 4}{h} = \lim_{h\to 0} \frac{4h + h^2}{h}.$$

Again, because h is approaching but never reaching 0 (the place where the denominator is 0), we can cancel the common factor of h:

$$\lim_{h\to 0} \frac{(2+h)^2 - 4}{h} = \lim_{h\to 0} (4+h) = 4.$$