Define Continuous:

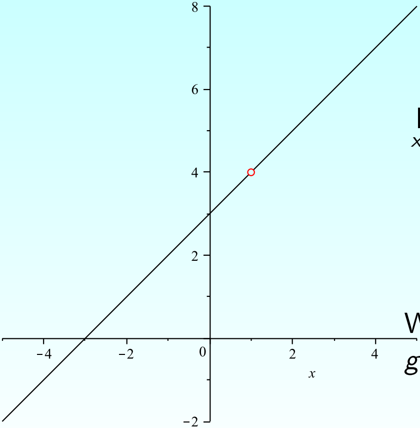
Definition: A function f is **continuous at $\mathbf{x = a}$** when

$(i) f(a)$ is defined $\quad (ii) \lim_{x \to a} f(x)$ exists $\quad$ and $(iii) \lim_{x \to a} f(x) = f(a)$.

Otherwise, f is said to be **discontinuous at $\mathbf{x = a}$**.

Reading Question # 3 (Example 1.4.2)

Let $g(x) = \begin{cases} \dfrac{x^2 + 2x - 3}{x - 1} & \text{if } x \neq 1, \\ a & \text{if } x = 1 \end{cases}$ for some real number a.



$$\begin{aligned} \lim_{x\to 1} g(x) &= \lim_{x\to 1} \frac{x^2 + 2x - 3}{x - 1} \\ &= \lim_{x\to 1} \frac{(x+3)(x-1)}{x-1} \\ &= \lim_{x\to 1} (x+3) = 4 \end{aligned}$$

Will $g(x)$ be continuous if we define $g(1) = 7$?

Reading Question # 4

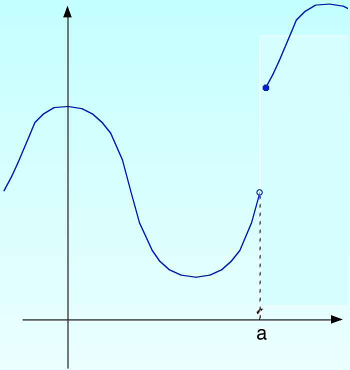


Figure 1.22b

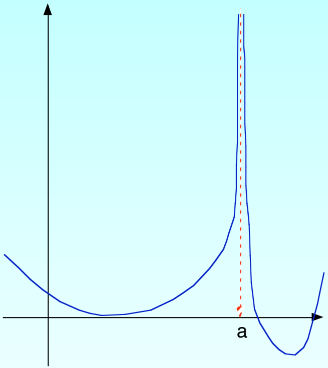
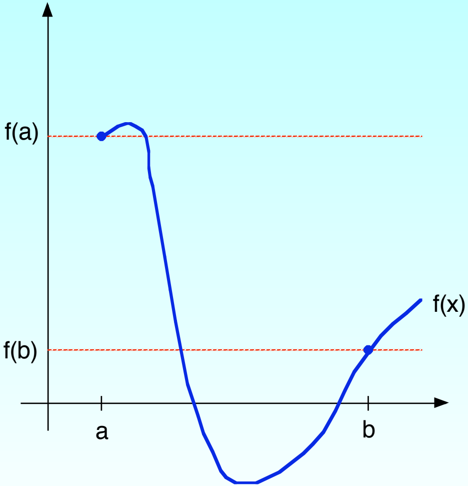
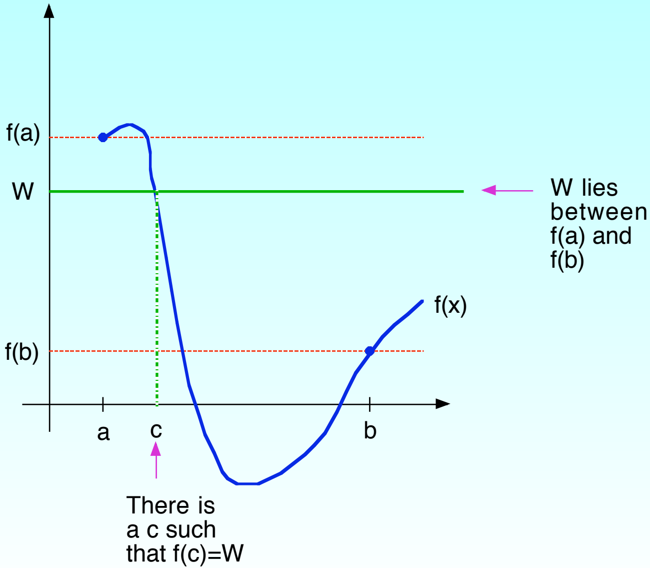


Figure 1.22d

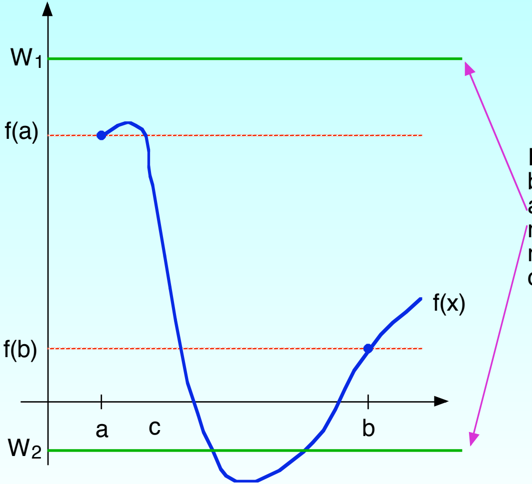
Illustrating the IVT:



Illustrating the IVT:



Illustrating the IVT:



If W doesn't lie between f(a) and f(b), there may or may not be such a c.

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	$[0.5, 0.75]$

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	$[0.5, 0.75]$
$[0.5, 0.75]$	> 0	< 0	0.625	$0.186 > 0$	

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	$[0.5, 0.75]$
$[0.5, 0.75]$	> 0	< 0	0.625	$0.186 > 0$	$[0.625, 0.75]$

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	$[0.5, 0.75]$
$[0.5, 0.75]$	> 0	< 0	0.625	$0.186 > 0$	$[0.625, 0.75]$
$[0.625, 0.75]$	> 0	< 0	0.6875	$0.085 > 0$	

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	$[0.5, 0.75]$
$[0.5, 0.75]$	> 0	< 0	0.625	$0.186 > 0$	$[0.625, 0.75]$
$[0.625, 0.75]$	> 0	< 0	0.6875	$0.085 > 0$	$[0.6875, 0.75]$

The IVT and the Bisection Method:

Find a zero of $f(x) = \cos(x) - x$:

Interval $[a, b]$	$f(a)$	$f(b)$	mid point	$f(\text{midpoint})$	Which $\frac{1}{2}$-interval?
$[0, 1]$	> 0	< 0	0.5	$0.378 > 0$	$[0.5, 1]$
$[0.5, 1]$	> 0	< 0	0.75	$-0.018 < 0$	$[0.5, 0.75]$
$[0.5, 0.75]$	> 0	< 0	0.625	$0.186 > 0$	$[0.625, 0.75]$
$[0.625, 0.75]$	> 0	< 0	0.6875	$0.085 > 0$	$[0.6875, 0.75]$
$[0.6875, 0.75]$	> 0	< 0	0.71875	$0.034 > 0$	$[0.71875, 0.75]$
$[0.71875, 0.75]$	> 0	< 0	0.734375	$0.008 > 0$	$[0.734375, 0.75]$
$[0.734375, 0.75]$	> 0	< 0	0.7421875	$-0.005 > 0$	$[0.734375, 0.742$

In Class Work

1. Let $f(x) = 14\sin(3x) + 2x^2 - 4x^3$. Use the IVT to show that $f(x)$ has a root between $x = -2$ and $x = 2$.

2. (a) Let $f(x) = \dfrac{1}{x-2}$. Use the IVT to show that $f(x)$ has a root between $x = 1$ and $x = 3$.

   (b) Find the exact value of the root by solving $f(x) = 0$. What goes wrong?

   (c) Reconcile your answers to parts (a) and (b).

Solutions:

1. Let $f(x) = 14\sin(3x) + 2x^2 - 4x^3$. Use the IVT to show that $f(x)$ has a root between $x = -2$ and $x = 2$.

$$\begin{aligned} f(-2) &= 14\sin(-6) + 8 - 4(-8) = 14\sin(-6) + 8 + 32 > 0 \\ f(2) &= 14\sin(6) + 8 - 32 < 0 \end{aligned}$$

Because f is continuous on $[-2, 2]$ and because 0 is between $f(-2)$ and $f(2)$, there must be some $c \in [-2, 2]$ such that $f(c) = 0$. Therefore f has a root between $x = -2$ and $x = 2$.

Solutions:

2. Let $f(x) = \dfrac{1}{x-2}$.

   (a) Use the IVT to show that $f(x)$ has a root between $x = 1$ and $x = 3$.

   $$f(1) = -1 \qquad f(3) = 1$$

   Since $f(1) < 0$ and $f(3) > 0$, it seems that f has a root between $x = 1$ and $x = 3$.

   (b) Find the exact value of the root by solving $f(x) = 0$. What goes wrong?

   $$\frac{1}{x-2} = 0 \Longrightarrow (x-2)\left(\frac{1}{x-2}\right) = (x-2)(0) \Longrightarrow 1 = 0!!$$

   Nonsensical conclusion $\Longrightarrow$ Original set-up must have made no sense $\Longrightarrow$ there is no root!

2. (continued)

   (c) Reconcile your answers to parts (a) and (b).

   How can there not be a root – we used the IVT to show a root must exist!!

   But did we? Did we ever check to see whether the hypotheses of the theorem apply?

   Is $f(x)$ continuous on $[1, 3]$?

   No – $f(x) = \dfrac{1}{x-2}$ is not defined at $x = 2$.