Expanding on Reading Question #1

Why $\lim_{x \to -3} \frac{1}{(x+3)^4} = 0$:

From the right:

x	$(x+3)^4$	$\frac{1}{(x+3)^4}$
-2.9	$(0.1)^4 = 10^{-4}$	10^4
-2.99	$(0.01)^4 = 10^{-8}$	10^8
-2.999	$(0.001)^4 = 10^{-12}$	10^{12}
-2.9999	$(0.0001)^4 = 10^{-16}$	10^{16}

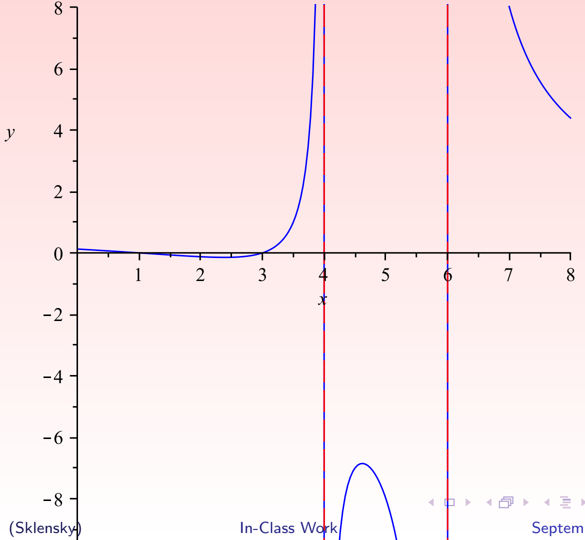
From the left:

x	$(x+3)^4$	$\frac{1}{(x+3)^4}$
-3.1	$(-0.1)^4 = 10^{-4}$	10^4
-3.01	$(-0.01)^4 = 10^{-8}$	10^8
-3.001	$(-0.001)^4 = 10^{-12}$	10^12
-3.0001	$(-0.0001)^4 = 10^{-16}$	10^{16}

Dividing a non-zero number by a small number results in a large number - the smaller the divisor, the larger the result!

Graph for Reading Question #2

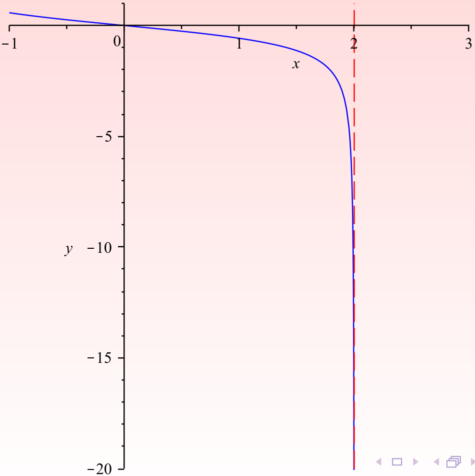
$$f(x) = \frac{(x-1)(x-3)}{(x-6)(x-4)}$$



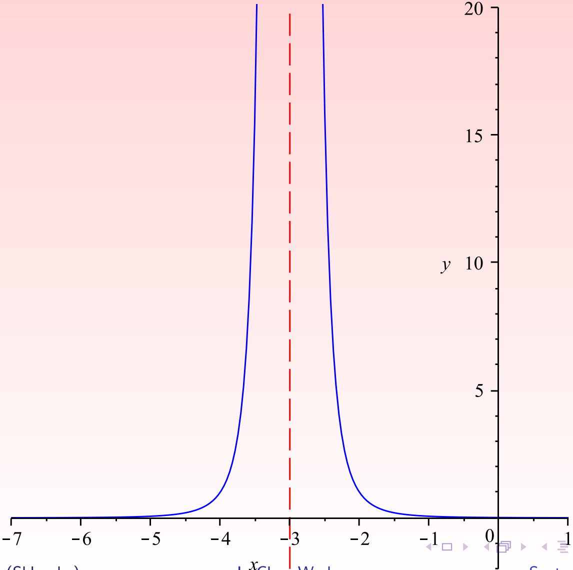
Example: $f(x) = \dfrac{-x}{\sqrt{4-x^2}}$

Domain: $x \leq 2$

$\lim_{x \to 2} f(x) = \lim_{x \to 2^-} f(x) = -\infty$



Graph of $f(x) = \dfrac{1}{(x+3)^4}$



In Class Work

1. Determine each limit.

   (a) $\lim_{x \to 2} \dfrac{x-4}{x^2 - 4x + 4}$

   (b) $\lim_{x \to 0} e^{-2/x}$

   (c) $\lim_{x \to \infty} \dfrac{x^3 - 2}{3x^2 + 4x - 1}$

2. Determine all horizontal and vertical asymptotes of $f(x) = \dfrac{3x^2+1}{x^2 - 2x - 3}$. For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the asymptote.

Solutions to In Class Work

1. Determine each limit.

   (a) $\lim\limits_{x \to 2} \dfrac{x-4}{x^2-4x+4}$

   - As $x \to 2$, $x - 4 \to -2$
   - As $x \to 2$, $x^2 - 4x + 4 = (x-2)(x-2) \to 0$

   Thus the limit does not exist, but in what way?

   - As $x \to 2^-$, $f(x) \to \frac{-}{(-)(-)} = -\infty$
   - As $x \to 2^+$, $f(x) \to \frac{-}{(+)(+)} = -\infty$

   Thus $\lim\limits_{x \to 2} \dfrac{x-4}{x^2-4x+4} = -\infty$.

Solutions to In Class Work

1. Determine each limit.

   (b) $\lim_{x \to 0} e^{-2/x}$

   - Rewrite as $\lim_{x \to 0} \frac{1}{e^{2/x}}$.
   - As $x \to 0^-$, $\frac{2}{x} \to -\infty$.

     As $x \to 0^+$, $\frac{2}{x} \to \infty$.

     $\lim_{x \to 0} \frac{2}{x}$ d.n.e.
   - $e^{\text{really large positive number}}$ is really large
     $e^{\text{really large negative number}}$ is very small

     Therefore totally different things are happening to our function, depending on what side we're approaching 0 from.

     $\lim_{x \to 0} \frac{1}{e^{2/x}}$ does not exist.

Solutions to In Class Work

1. Determine each limit.

   (c) $\lim_{x \to \infty} \dfrac{x^3 - 2}{3x^2 + 4x - 1}$

   Rational function.

   - Degree of numerator $=3$
   - Degree of denominator $= 2$
   - Degree of numerator $>$ degree of denominator
   - Numerator approaches infinity much faster than denominator
   - Coefficient of x^3 in numerator is 1, coefficient of x^2 in denominator is 3 – both are positive.

$$\lim_{x \to \infty} \frac{x^3 - 2}{3x^2 + 4x - 1} = \infty$$

Solutions to In Class Work

2. Determine all horizontal and vertical asymptotes of $f(x) = \dfrac{3x^2+1}{x^2-2x-3}$. For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the asymptote.

   Vertical Asymptotes:

   - $f(x) = \dfrac{3x^2+1}{x^2-2x-3} = \dfrac{3x^2+1}{(x-3)(x+1)}$
   - Thus vertical asymptotes exist at $x = 3$ and $x = -1$.
     - $x = 3$:
       - As $x \to 3^-$, we have $\dfrac{+}{(-)(+)} = -$, so $\lim\limits_{x\to 3^-} f(x) = -\infty$
       - As $x \to 3^+$, we have $\dfrac{+}{(+)(+)} = +$, so $\lim\limits_{x\to 3^+} f(x) = \infty$

Solutions to In Class Work

2. Determine all horizontal and vertical asymptotes of $f(x) = \dfrac{3x^2+1}{x^2-2x-3}$. For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the asymptote.

   Vertical Asymptotes, continued

   - Thus vertical asymptotes exist at $x = 3$ and $x = -1$.
     - $x = -1$:
       - As $x \to -1^-$, we have $\dfrac{+}{(-)(-)} = +$, so $\lim\limits_{x \to -1^-} f(x) = \infty$
       - As $x \to -1^+$, we have $\dfrac{+}{(-)(+)} = -$, so $\lim\limits_{x \to -1^+} f(x) = -\infty$

2. Determine all horizontal and vertical asymptotes of $f(x) = \dfrac{3x^2+1}{x^2-2x-3}$. For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the asymptote.

Horizontal Asymptotes:

- Need to find $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$.
- Degree of numerator=2
- Degree of denominator =2
- Numerator and denominator have the same degree
- Neither dominates over the other
- Limit at ∞ will be ratio of leading coefficients
- Coefficient of x^2 in numerator is 3, coefficient of x^2 in denominator is 1
- $\lim_{x\to\infty} f(x) = \dfrac{3}{1} = 3$
- As for $-\infty$, just need to check if anything becomes negative.
- Because highest degree of both is even, negative aren't introduced, so $\lim_{x\to-\infty} f(x) = \dfrac{3}{1} = 3$

2. Determine all horizontal and vertical asymptotes of $f(x) = \dfrac{3x^2+1}{x^2-2x-3}$. For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the asymptote.

Conclusion: $f(x)$ has a

- horizontal asymptote at $y = 3$
- vertical asymptote at $x = -1$. $\lim\limits_{x \to -1^-} f(x) = \infty$, $\lim\limits_{x \to -1^+} f(x) = -\infty$.
- 2nd vertical asymptote at $x = 3$. $\lim\limits_{x \to 3^-} f(x) = -\infty$, $\lim\limits_{x \to 3^+} f(x) = \infty$.

2. Determine all horizontal and vertical asymptotes of $f(x) = \dfrac{3x^2+1}{x^2-2x-3}$. For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the asymptote.

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