Slope of Secant Line =Average Rate of Change

If we are looking at the graph of $y = f(x)$, then

$$\text{slope of secant line from } x = a \text{ to } x = b = \frac{\text{rise}}{\text{run}}$$
$$= \frac{\Delta y}{\Delta x}$$
$$= \boxed{\frac{f(b) - f(a)}{b - a}}$$

If we are looking at an object whose position is given by $f(t)$, then

$$\text{average r.o.c. from } t = a \text{ to } t = b = \frac{\text{change in position}}{\text{time}}$$
$$= \frac{f(t_{\text{final}}) - f(t_{\text{initial}})}{t_{\text{Final}} - t_{\text{initial}}}$$
$$= \boxed{\frac{f(b) - f(a)}{b - a}.}$$

From Average RofC to Instantaneous RofC

$$\text{average r.o.c. from } t = a \text{ to } t = b = \frac{f(b) - f(a)}{b - a}.$$

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We discovered: the smaller the time interval, that is, the closer $t = b$ is to $t = a$, the better job the average rate of change does at approximating the instantaneous rate of change at $t = a$.

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We discovered: the smaller the time interval, that is, the closer $t = b$ is to $t = a$, the better job the average rate of change does at approximating the instantaneous rate of change at $t = a$.

In other words, we were heading toward the notion of a limit!

$$\text{inst r.o.c. at } t = a \text{ is } \lim_{t \to a} \frac{f(t) - f(a)}{t - a}$$

From Slope of Secant Line to Slope of Tangent Line

slope of secant line from $x = a$ to $x = b = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$.

From Slope of Secant Line to Slope of Tangent Line

slope of secant line from $x = a$ to $x = b = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$.

Again, we decided that the closer b is to a, the better job the slope of the secant line does of approximating the slope of the curve itself at $x = a$.

From Slope of Secant Line to Slope of Tangent Line

$$\text{slope of secant line from } x = a \text{ to } x = b = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}.$$

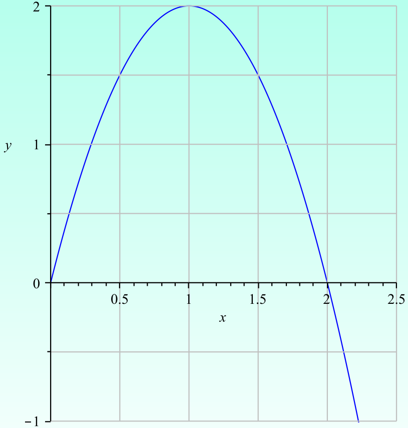
Again, we decided that the closer b is to a, the better job the slope of the secant line does of approximating the slope of the curve itself at $x = a$.

That is, once again we were heading toward the idea of a limit:

$$\text{slope of tangent line at } x = a = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

In Class Work

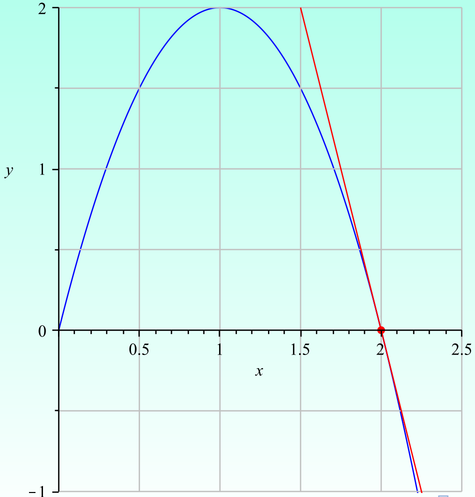
Graph of
$f(x) = -2(x-1)^2 + 2$



1. Sketch the line tangent to $f(x)$ at $x = 2$, and then estimate the slope of that tangent line.
2. Use the limit definition to find the exact value of this slope.
3. Find the equation of the line tangent to $f(x)$ at $x = 2$.

Solutions

1. Sketch the line tangent to $f(x)$ at $x = 2$, and then estimate the slope of that tangent line.



Solutions

2. Use the limit definition to find the exact value of this slope.

$$
\begin{aligned}
m_{tan} &= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\
&= \lim_{h\to 0} \frac{\left[-2\big((2+h)-1\big)^2+2\right]-\left[-2(2-1)^2+2\right]}{h} \\
&= \lim_{h\to 0} \frac{\left[-2(1+h)^2+2\right]-\left[-2+2\right]}{h} \\
&= \lim_{h\to 0} \frac{\left[-2(1+2h+h^2)+2\right]-\left[0\right]}{h} \\
&= \lim_{h\to 0} \frac{\left[-2-4h-2h^2+2\right]}{h} = \lim_{h\to 0} \frac{-4h-2h^2}{h} \\
&= \lim_{h\to 0} (-2h-4) = \boxed{-4}
\end{aligned}
$$

In Class Work

1. Find the equation of the line tangent to $f(x)$ at $x = 2$.

   Need: a point and the slope.

   Slope=-4

   Point: Use the point of tangency.
   At $x = 2$, $f(x) = -2(2-1)^2 + 2 = 0$, so the point of tangency is $(2, 0)$.

   Tangent Line:

   $$y - y_1 = m(x - x_1) \Longrightarrow y - 0 = -4(x-2) \Longrightarrow y = -4x + 8.$$